OCR

Tuesday 24 June 2014 – Morning

A2 GCE MATHEMATICS (MEI)

4773/01 Decision Mathematics Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

1 The owner of a grocery store aims to keep 150 cartons of UHT milk in stock. At the end of each working day the store keeper counts the stock and places an order. If the stock is less than 150 he orders the difference between 150 and the stock. If the stock is 150 or more then he does not order any. Orders take one working day to be processed and are then delivered at the start of the next working day after that. The store is closed on Sundays.

At the end of Friday the store keeper ordered 35 cartons of UHT milk. These arrived on Monday morning.

On Saturday he received cartons from Thursday's order and sold 50 cartons. He ended the day with 130 cartons in stock and ordered 20.

On Monday he received 35 from Friday's order and again sold 50. So at the close of trading he had 115 in stock, and he ordered 35.

(i) Build a spreadsheet to model u_n, the number of cartons in stock at the beginning of day n before receiving that day's delivery, starting with Monday as day 1. You will need to model the possibility of him ordering no cartons, and you need to show your formulae.

Assuming that demand remains constant at 50 per day, report on how stock levels behave in the long run. **[5]**

(ii) For this scenario the owner does not ever order 0 cartons. So, ignoring the possibility that the stock might exceed 150 at the end of a day, construct a recurrence relation to model the scenario (ie demand constant at 50 per day). **[2]**

(iii) Find the auxiliary equation for your recurrence relation, and show that it has no real roots.

Explain how you could have seen that the auxiliary equation would have no real roots by inspecting your spreadsheet. **[3]**

The owner's daughter suggests that her father tries ordering 50 cartons plus a fraction, α, of the difference between 150 and the closing stock, $0 < \alpha < 1$, for closing stocks less than or equal to 150. (You are not required to model the possibility of closing stocks greater than 150.) So on Saturday he would have ended the day with 130 cartons in stock and have ordered $50 + 20\alpha$. On Monday he would have had 115 in stock at the close of trading, and would have ordered $50 + 35\alpha$.

(iv) Construct the recurrence relation for u_n, the number of cartons in stock at the beginning of day n, with Monday being day 1 and assuming that demand remains constant at 50 per day. Show that for $\alpha = 0.25$, the recurrence relation's auxiliary equation has one real root. **[4]**

(v) Add a model of this scenario to your spreadsheet. Use your spreadsheet to demonstrate that for $\alpha = 0.25$ and for demand constant at 50, the solution to the recurrence relation is $u_n = 60(0.5)^n - 100n(0.5)^n + 150$. **[2]**

(vi) The recurrence relation given in part **(v)** is a mathematical approximation to a realistic ordering policy since it allows non-integer values. Produce a spreadsheet model with $\alpha = 0.25$, but in which order quantities are to the nearest whole number. **[1]**

(vii) Use your spreadsheet from part **(vi)** to investigate what increased level of demand leads to stock running out. **[1]**

2 A cable car installation at a ski resort has two cabins connected by the cable. Skiers board the lower cabin until it is full. It then travels up whilst the empty cabin travels down from the top station.

The cabins can each hold 160 skiers. The journey time from bottom to top is 4 minutes. So the system can move 2400 skiers per hour, or an average rate of 2 skiers every 3 seconds.

Skiers arrive at an average rate of 1 skier every 2 seconds at intervals given by the following distribution.

time interval (secs)	1	2	3	4
probability	0.4	0.3	0.2	0.1

(i) Build a spreadsheet simulation for loading and transporting 160 skiers, assuming that an empty cabin is waiting for the first skier. For each skier, record the time between his or her arrival and the time at which the cabin completes its journey, and find the mean of these times. This is an estimate of the "mean time in system". **[6]**

(ii) Explain why the assumption that an empty cabin is waiting for the first skier will also model the situation in which the first skier arrives just after a cabin has departed, assuming that skiers are arriving at a rate of 1 skier every 2 seconds. **[1]**

(iii) Run your simulation 10 times, recording the mean time in system for each of your 10 runs. **[2]**

(iv) Criticise the modelling assumptions. **[1]**

In a modernisation the cabins are replaced by cabins which each hold 120 skiers, but which can complete the journey in 3 minutes, so that the capacity is unchanged at 2400 skiers per hour.

(v) Simulate this system for 120 skiers, assuming that an empty cabin is waiting for the first skier. Repeat your simulation 10 times and compare this mean time in system to that in the old system. **[3]**

In fact, the new cabins are programmed to depart when full or at 4-minute intervals, whichever happens first.

(vi) Simulate this system, assuming that an empty cabin is waiting for the first skier. Explain how you dealt with the limit of no more than 4 minutes between departures.

Repeat your simulation 10 times, each time assuming that an empty cabin is waiting for the first skier. Record how many skiers were loaded in each simulation. Compare the mean time in system to that which you observed in part **(iii)**. **[4]**

(vii) When would it become necessary to modify the modelling assumption that an empty cabin is waiting for the first skier? **[1]**

3 **(i)** Put the following LP into LINDO format, run it, and explain what it does.

```
max   M
st    M < 65
      M < 37
      M < 19
      M < 54
      M < 23
end
```

[4]

The table gives the complete set of shortest distances between the vertices of a network.

	A	B	C	D	E
A	0	23	42	35	52
B	23	0	37	29	43
C	42	37	0	18	50
D	35	29	18	0	32
E	52	43	50	32	0

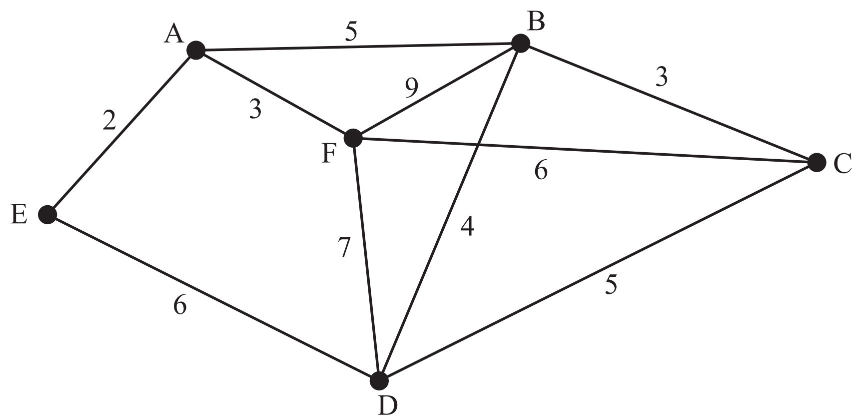
The following LP finds the minimum of the row maxima. The "R" variables give the row maxima. "M" gives the minimum of these, using the approach of part **(i)**. Maximising Y forces the R variables to be as small as possible and M to be as large as possible at the same time.

```
maximise   Y
st         Y=M–2R1–2R2–2R3–2R4–2R5
           M < R1
           M < R2
           M < R3
           M < R4
           M < R5
           R1 > 23
           R1 > 42
           R1 > 35
           R1 > 52
           R2 > 23
           R2 > 37
           R2 > 29
           R2 > 43
           R3 > 42
           R3 > 37
           R3 > 18
           R3 > 50
           R4 > 35
           R4 > 29
           R4 > 18
           R4 > 32
           R5 > 52
           R5 > 43
           R5 > 50
           R5 > 32
end
```

(ii) Put the LP into LINDO format, and run it. You will need to put the instruction "Free Y" on a line after your "End" statement, since Y will take a negative value.

Interpret the output. You should explain what gives the minimum of the row maxima, and how you can find which row contains this. **[6]**

John is designing a hot water system for his new house. The vertices on the network represent positions for taps. The arcs represent possible connections between taps. The weights represent the lengths in metres of pipes that would be needed to connect taps. John wants to position the hot water cylinder so that the maximum distance from it to a tap is as small as possible.



(iii) Explain the steps that you would go through to use LP to find the best vertex for the location of John's hot water cylinder. You do not need to produce or run an LP. **[5]**

(iv) Use inspection to find the four best vertices at which to position the cylinder, each with a maximum distance of 8 metres from other vertices. **[1]**

(v) Why might LP be used to solve real-world problems of this nature when they seem easy to do by inspection? **[1]**

(vi) Show that, in fact, the best position for the cylinder is not at a vertex. **[1]**

Question 4 begins on page 6

4 KPQ Logistics is a distribution company. It has two distribution centres, one in the north of the country (N) and one in the south (S). It serves producers of goods, taking deliveries into its distribution centres and delivering from the centres on to shops.

The company maintains a large database detailing the costs of delivery from producers to the company's distribution centres, and costs of delivery from the centres to customers.

The company can move up to 200 tonnes a week between its two centres at a cost of £5 per tonne.

HJS Manufacturers produce a chemical in its two factories, F1 and F2. It is sold by the tonne, mainly to 10 customers located in different parts of the country. HJS employs KPQ to distribute this chemical.

Next week's production schedule and orders are detailed below, as are the transport costs from factories to centres and from centres to customers.

	F1	F2
Next week's production (tonnes)	1000	750

Customer	A	B	C	D	E	F	G	H	I	J
Next week's orders (tonnes)	170	70	400	150	80	120	50	175	200	300

Transport costs (£ per tonne)

	F1	F2
N	12	7
S	5	15

	A	B	C	D	E	F	G	H	I	J
N	13	15	7	7	20	21	14	11	8	9
S	4	12	14	17	11	10	7	8	14	15

KPQ wishes to construct a transport plan for next week which will minimise its costs.

(i) Formulate this as an LP problem. **[10]**

(ii) Solve your LP, interpret the solution, and report the results for KPQ. **[6]**

(iii) Suggest two ways in which costs might be reduced further without incurring any capital expense. **[2]**

END OF QUESTION PAPER

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