OCR
Oxford Cambridge and RSA

Tuesday 9 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The displacement, x m, of a particle at time t s is given by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+8\frac{\mathrm{d}x}{\mathrm{d}t}+25x=0.$$

Initially the particle is at the origin and has a velocity of $\frac{1}{4}\,\mathrm{m\,s^{-1}}$.

(i) Find the particular solution for x. **[8]**

(ii) Find the maximum displacement of the particle from its initial position, giving your answer correct to 3 significant figures. **[4]**

(iii) Describe the behaviour of your solution for large values of t. **[1]**

In a different situation, an additional force is applied to the particle and the differential equation satisfied by x is

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+8\frac{\mathrm{d}x}{\mathrm{d}t}+25x=5\sin 5t.$$

(iv) Using the same initial conditions as in part **(i)**, find the new particular solution for x. **[10]**

(v) Describe the behaviour of your new solution for large values of t. **[1]**

2 The differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x}-ny=2x-1,$$

where n is a non-zero constant, is to be solved for $x>0$.

Firstly consider the case $n\neq 1$.

(i) Find the general solution for y in terms of x and n. **[8]**

(ii) For $n=-1$, find the equation of the solution curve that passes through the point $(2,0)$ and sketch the curve for $x>0$. **[4]**

Now consider the case $n=1$.

(iii) Find the general solution for y in terms of x. **[5]**

(iv) Show that the solution curve for which $y=0$ when $x=1$ has exactly one stationary point. **[3]**

Now consider the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x}-y=\frac{1}{\sqrt{2x-1}}.$$

(v) Use Euler's method, with a step length of 0.1 and initial conditions $y=0$ when $x=1$, to estimate y when $x=1.3$. The algorithm is given by $x_{r+1}=x_r+h$, $y_{r+1}=y_r+hy'_r$. **[4]**

3 The resistance to motion of a small test car of mass 20 kg is modelled differently according to the aerodynamic features of the bodywork being tested. The motion of the test car is studied as it moves in a horizontal straight line. In each trial, the car is initially at rest at A; at time t s its velocity is $v\,\mathrm{m\,s^{-1}}$ and its distance from A is x m. The only horizontal forces acting on the car are a driving force of 100 N and a varying resistance force of magnitude R N.

In the first trial, the resistance to motion is modelled by $R = 4v^2$.

(i) Write down and solve a differential equation to show that

$$v^2 = 25\left(1 - \mathrm{e}^{-\frac{2}{5}x}\right).$$

Find the value of v when $x = 10$. **[9]**

(ii) Find the value of t when $x = 10$. **[7]**

In the second trial, the resistance to motion is modelled by $R = 2v$.

(iii) Write down and solve a differential equation to find v in terms of t. State the terminal velocity of the car. **[7]**

(iv) Find the value of t in the second trial when the car's speed is equal to the value of v found in part **(i)**. **[1]**

4 Two species of small rodent, X and Y, compete for survival in the same environment. The populations of the species, at time t years, are x and y respectively and they are modelled by the simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2(x - y),$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{3}{8}\left(x - 80\mathrm{e}^{-\frac{1}{2}t}\right).$$

(i) Show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 2\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{3}{4}x = 60\mathrm{e}^{-\frac{1}{2}t}.$$

Find the general solution for x. **[10]**

(ii) Find the corresponding general solution for y. **[3]**

When $t = 0$, $x = 40$ and $y = 50$.

(iii) Find the particular solutions for x and y. **[4]**

(iv) Find the time T at which the model predicts that the rodents of species X will die out. Find the population of species Y predicted at this time. **[6]**

(v) Comment on the suitability of the model for times greater than T. **[1]**

END OF QUESTION PAPER