OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4757**: Further Applications of Advanced Mathematics

Advanced GCE

Mark Scheme for June 2015

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

1	(i)	Any point is $\begin{pmatrix}8\\25\\43\end{pmatrix}+\lambda\begin{pmatrix}4\\15\\25\end{pmatrix}$ $=((8+4\lambda),(25+15\lambda),(43+25\lambda))$	M1	Finding AB in terms of λ	
		$\text{AB}=((8+4\lambda),(25+15\lambda),(43+25\lambda))-(2,5,4)$ $=((6+4\lambda),(20+15\lambda),(39+25\lambda))$	A1		
		Distance AB $=\sqrt{(6+4\lambda)^2+(20+15\lambda)^2+(39+25\lambda)^2}=15$	M1		
		$\Rightarrow 866\lambda^2+2598\lambda+1732=0$	A1		
		$\Rightarrow \lambda^2+3\lambda+2=0$ $\Rightarrow \lambda=-1,\ -2$ $\Rightarrow$ B $(4,10,18)$, C $(0,-5,-7)$	B1 A1	For B For C	B1 can also be given for verifying AB = 15 and showing B is on line
			6		
	(ii)	$\text{AC}=[2,10,11]$	B1	Or any vector in plane other than BC	
		$\mathbf{n}=\begin{pmatrix}2\\10\\11\end{pmatrix}\times\begin{pmatrix}4\\15\\25\end{pmatrix}=\begin{pmatrix}85\\-6\\-10\end{pmatrix}$	M1	Suitable vector product or other method for finding normal	
		$\Rightarrow 85x-6y-10z=c$	A1		
		Sub one value to give c $\Rightarrow 85x-6y-10z=100$	A1		
			4		

	(iii)	This plane contains the line BC and **n** $\mathbf{n'} = \begin{pmatrix} 85 \\ -6 \\ -10 \end{pmatrix} \times \begin{pmatrix} 4 \\ 15 \\ 25 \end{pmatrix} = \begin{pmatrix} 0 \\ -2165 \\ 1299 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \\ 3 \end{pmatrix}$ $\Rightarrow 5y - 3z = c$ Sub one value to give c $\Rightarrow 5y - 3z + 4 = 0$	**M1** **A1** **M1** **A1**	Appropriate vector product **oe** Allow uncancelled vector Must be seen!	
			4		
	(iv)	$\mathbf{BC} = [-4, -15, -25]$ $\quad \mathbf{AD} = [-1, -4, -1]$ $\mathbf{BC} \times \mathbf{AD} = \begin{bmatrix} 4 & 15 & 25 \\ 1 & 4 & 1 \end{bmatrix} = [-85, 21, 1]$ $\lvert \mathbf{BC} \times \mathbf{AD} \rvert = \sqrt{85^2 + 21^2 + 1} = \sqrt{7667}$ **AC =** $[2, 10, 11]$ $\text{Distance} = \left\lvert \frac{(\mathbf{AC}).(\mathbf{BC} \times \mathbf{AD})}{\sqrt{7667}} \right\rvert$ $= \left\lvert \frac{-170 + 210 + 11}{\sqrt{7667}} \right\rvert = \frac{51}{\sqrt{7667}}$	**B1** **M1** **M1** **A1** **B1** **M1** **A1**	 Finding vector product Finding magnitude, pythagoras must be seen N.b. Answer given Other vectors possible Accept 0.582.....	
			7		

	(v)	A (2, 5, 4), B(4, 10, 18), C(0,-5, -7), D(1, 1, 3) **AB** = $[2,5,14]$ **AC** = $[-2,-10,-11]$ **AD** = $[-1,-4,-1]$ Volume = $\left\|\frac{1}{6}(\mathbf{AB}\bullet(\mathbf{CA}\times\mathbf{DA})\right\|$ $\mathbf{AC}\times\mathbf{AD}=\begin{bmatrix}-2 & -10 & -11\\ -1 & -4 & -1\end{bmatrix}=[-34,9,-2]$ $=\left\|\frac{1}{6}((2,5,14)\bullet(-34,9,-2))\right\|=\left\|\frac{1}{6}(-68+45-28)\right\|$ $=\frac{51}{6}=\frac{17}{2}$	**M1** **A1** **A1**	Formula for volume Vector product	
			3		

2	**(i)**	$z = 3x^2 - 12xy + 2y^3 + 60$ $\frac{\partial z}{\partial x} = 6x - 12y = 0 \Rightarrow x = 2y$ $\frac{\partial z}{\partial y} = -12x + 6y^2 = 0 \Rightarrow y^2 = 2x = 4y$ $\Rightarrow y = 0$ or $4. \quad \Rightarrow x = 0$ or $8.$ $\Rightarrow z = 60$ or -4 Stationary points at $A(8, 4, -4)$, and $B(0, 0, 60)$	**M1** **A1** **M1** **B1** **A1**	For finding both partials and setting = 0 Both Solving simultaneously to get x or y Also by verification	
			5		
	(ii)(*A*)	$z = 3x^2 - 12xy + 2y^3 + 60$ Substitute $x = 8 + h, \quad y = 4 + k$ $\Rightarrow z_p = 3(8+h)^2 - 12(8+h)(4+k) + 2(4+k)^3 + 60$ $= -4 + 3(h^2 - 4hk + 4k^2) + 12k^2 + 2k^3$ $= -4 + 3(h - 2k)^2 + 2k^2(k+6)$	**M1** **M1** **A1**	For substitution and expansion For splitting $24k^2$	
			3		
	(ii)(*B*)	For all values of h and k, $(h - 2k)^2 > 0$ and $2k^2(k + 6) > 0$ providing $k > -6$ So $z > -4$ for all values of x and y close to A So is local minimum.	**M1** **M1** **A1**	 Or for small k	
			3		

	(ii)(*C*)	When $x = 0,\ z = 2y^3 + 60$ and so either side of (0, 0, 60) the value of z will be greater or less than 60. When $y = 0,\ z = 3x^2 + 60$ and so either side of (0, 0, 60) the value of z will always be greater than 60. So B is a saddle point.	**M1** **A1** **A1** **A1**	For obtaining functions For pt of inflexion - can be by sketch For minimum - can be by sketch	
			4		
	(iii)	At (1, 1, 53), $\frac{\partial z}{\partial x} = 6x - 12y = -6$ $\frac{\partial z}{\partial y} = -12x + 6y^2 = -6$ Equation of tangent plane is: $z - 53 = \frac{\partial z}{\partial x}(x-1) + \frac{\partial z}{\partial y}(y-1)$ $z - 53 = -6x - 6y + 12$ $\Rightarrow 6x + 6y + z = 65$	**M1** **A1** **M1** **A1**	Finding values of derivatives Eqn of plane ag	Or $-6x - 6y - z = c$
			4		
	(iv)	$\frac{\partial z}{\partial x} = -6$ and $\frac{\partial z}{\partial y} = -6$ $\Rightarrow x = 2y - 1$ and $y^2 = 2x - 1$ $\Rightarrow y^2 - 4y + 3 = 0$ $\Rightarrow y = 1, 3$ $\Rightarrow$ Coordinates of R $(5, 3, 9)$	**M1** **A1** **M1** **A1** **A1**	Put partial derivatives = –6 Both correct Solve simultaneously Both values	
			5		

3	(i)	$x = a\cos\theta,\ x' = -a\sin\theta,\ x'' = -a\cos\theta$ $y = b\sin\theta,\ y' = b\cos\theta,\ y'' = -b\sin\theta$	B1	derivatives	
		$r = \dfrac{\left((x')^2+(y')^2\right)^{3/2}}{x'y''-x''y'} = \dfrac{\left(a^2\sin^2\theta+b^2\cos^2\theta\right)^{3/2}}{-a\sin\theta.-b\sin\theta--a\cos\theta.b\cos\theta}$	M1	Apply formula (or for κ)	
		$= \dfrac{\left(a^2\sin^2\theta+b^2\cos^2\theta\right)^{3/2}}{ab}$	M1	Set $\theta = 0$	
		At A, $\theta = 0 \Rightarrow r = \dfrac{\left(a^2\sin^2 0+b^2\cos^2 0\right)^{3/2}}{ab}$	A1	unsimplified	
		$= \dfrac{\left(b^2\right)^{3/2}}{ab} = \dfrac{b^2}{a}$	A1	ag	
		The centre is on the x-axis r less than a	M1		
		So centre of curvature is at $\left(a-\dfrac{b^2}{a},0\right)$ i.e. $\left(\dfrac{a^2-b^2}{a},0\right)$	A1		
			7		
	(ii)	Radius = $\dfrac{a^2}{b}$	B1		
		Centre is at $\left(0, b-\dfrac{a^2}{b}\right)$ i.e. $\left(0,\dfrac{b^2-a^2}{b}\right)$	B1		
			2		
	(iii)	$\dfrac{a^2}{b} = \dfrac{b^2}{a} \Rightarrow a = b$ The ellipse is a circle OR the centre of curvature for both points (and all points) is at (0, 0)	B1		
			1		

	(iv)	$s=\int_0^{\pi/2}\sqrt{\left(\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2\right)}\,d\theta$	M1	Applying formula	
		$=\int_0^{\pi/2}\sqrt{\left((a\sin\theta)^2+(b\cos\theta)^2\right)}\,d\theta$			
		$=\int_0^{\pi/2}\sqrt{\left(a^2\sin^2\theta+b^2\cos^2\theta\right)}\,d\theta$	A1		
		$=\int_0^{\pi/2}\sqrt{\left(b^2\sin^2\theta+b^2\cos^2\theta+(a^2-b^2)\sin^2\theta\right)}\,d\theta$	M1	Eliminate $\cos\theta$	
		$=\int_0^{\pi/2}\sqrt{\left(b^2+(a^2-b^2)\sin^2\theta\right)}\,d\theta$	A1		
		$=b\int_0^{\pi/2}\sqrt{\left(1+\frac{(a^2-b^2)}{b^2}\sin^2\theta\right)}\,d\theta$			
		$\Rightarrow\lambda^2=\frac{(a^2-b^2)}{b^2}$	A1		
		When $a=b$, $\lambda=0$			
		$\Rightarrow s=b\int_0^{\pi/2}d\theta=\frac{1}{2}b\pi$	A1		
		This is a quarter of the circumference of a circle	A1	Or arc length of part of circle	
			7		

	(v)	Centre of curvature is at $\left(a\cos\theta-\rho\sin\psi, b\sin\theta+\rho\cos\psi\right)$ where $\rho=\frac{\left(a^2\sin^2\theta+b^2\cos^2\theta\right)^{3/2}}{ab}$ giving $x=a\cos\theta-\frac{\left(a^2\sin^2\theta+b^2\cos^2\theta\right)^{3/2}}{ab}.\frac{b\cos\theta}{\left(a^2\sin^2\theta+b^2\cos^2\theta\right)^{1/2}}$ $\Rightarrow ax=\cos\theta\left(a^2-\left(a^2\sin^2\theta+b^2\cos^2\theta\right)\right)$ $=\cos\theta\left(a^2-b^2\right)\cos^2\theta=\cos^3\theta\left(a^2-b^2\right)$ Similarly $by=\sin^3\theta\left(b^2-a^2\right)$ $\Rightarrow\left(\frac{ax}{a^2-b^2}\right)^{2/3}+\left(\frac{by}{a^2-b^2}\right)^{2/3}=1$ **oe**	**M1** **M1** **A1** **A1** **A1** **M1** **A1**	Parametric equations Deal with ψ i.e. $(ax)^{2/3}+(by)^{2/3}=\left(a^2-b^2\right)^{2/3}$	Or find equation of normal Or partial diffn of normal Eqn for x or y
			7		

4	**(i)**	The set is closed. i.e. $\begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix}\begin{pmatrix} c & d \\ 0 & \frac{1}{c} \end{pmatrix} = \begin{pmatrix} ac & ad + \frac{b}{c} \\ 0 & \frac{1}{ac} \end{pmatrix}$ Identity is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or m(1,0) Each element has an inverse Inverse of $\begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix}$ is $\begin{pmatrix} \frac{1}{a} & -b \\ 0 & a \end{pmatrix}$ Which is in M	**M1** **A1** **A1** **B1** **M1** **A1** **A1**	Attempt to demonstrate closure Two general and different matrices Correct product therefore shows closure Attempt to demonstrate inverse A general matrix and its inverse	
			7		
	(ii)	e.g. product $\begin{pmatrix} c & d \\ 0 & \frac{1}{c} \end{pmatrix} \times \begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix} = \begin{pmatrix} ac & cb + \frac{d}{a} \\ 0 & \frac{1}{ac} \end{pmatrix}$ OR one numeric example. e.g. $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 0 & .5 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 0 & .5 \end{pmatrix}$ but $\begin{pmatrix} 2 & 3 \\ 0 & .5 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 7 \\ 0 & .5 \end{pmatrix}$ So No	**M1** **A1** **A1**	Demonstration by multiplying 2 matrices each way round (one way might be quoted from (i)) 2 correct (and different) products Dep on previous 2 marks	
			3		

	(iii)	e.g. $\begin{pmatrix} k & b \\ 0 & \frac{1}{k} \end{pmatrix}\begin{pmatrix} k & c \\ 0 & \frac{1}{k} \end{pmatrix} = \begin{pmatrix} k^2 & ..... \\ 0 & \frac{1}{k^2} \end{pmatrix}$ This is only in the set N_k if $k^2 = k$ Given $k \neq 0 \Rightarrow k = 1$ only	**M1** **A1** **A1** **A1**	Multiplying 2 matrices in N (Allow matrices the same) Sight of k^2	
			4		
	(iv)	$m(1,1)^2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$	**M1** **A1**		
			2		
	(v)	$\begin{matrix} & m(1,0) & m(1,1) & m(1,2) & m(1,3) \\ m(1,0) & m(1,0) & m(1,1) & m(1,2) & m(1,3) \\ m(1,1) & m(1,1) & m(1,2) & m(1,3) & m(1,0) \\ m(1,2) & m(1,2) & m(1,3) & m(1,0) & m(1,1) \\ m(1,3) & m(1,3) & m(1,0) & m(1,1) & m(1,2) \end{matrix}$	**B1** **B3**	m(1,0) the identity −1 each error	
			4		
	(vi)	Group table for R OR Any argument that states that: a, b, c have order 2 **B1** reason for this **B1** but only one element of P has order 2 **B1** So No	**B3** **B1**	−1 each error Dependent on previous B3	
			4		

5	(i)	$$\mathbf{P} = \begin{matrix} & A & B & C \\ A & 0 & 1/3 & 0 \\ B & 1/2 & 1/3 & 1/2 \\ C & 1/2 & 1/3 & 1/2 \end{matrix}$$	B2	B1 for two out of three columns correct	
			2		
	(ii)(*A*)	$\mathbf{P}^3 p$ $$\begin{pmatrix} 0.138\dot{8} & - & - \\ - & - & - \\ - & - & - \end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} \text{ gives } 0.138\dot{8}\left(=\frac{5}{36}\right)$$	M1 A1 A1	Cube P Sight of matrix soi	Allow M1 for P^4
			3		
	(ii)(*B*)	$P = M^5 p$ $$\mathbf{P}^5 = \begin{pmatrix} \ldots & - & - \\ - & 0.4285 & - \\ - & - & 0.4286 \end{pmatrix}$$ $p = 0.5 \times 0.4285 + 0.5 \times 0.4286$ $= 0.4286$	M1 M1 A1 A1	Using diagonal elements from P^5 Using probabilities from 2nd day Ft Cao	
			4		
	(iii)	0.143, 0.429, 0.429 $\left(=\frac{1}{7}, \frac{3}{7}, \frac{3}{7}\right)$ Over a long period these are the probabilities that on any day at random the inspector is at these factories	M1 B1 A1 B1	Obtaining equations Sight of $x + y + z = 1$ soi	Or **M1** considering P^n where n is large.(10 or more) **A2** for probabilities, A1 one error
			4		

	(iv)	$\mathbf{Q} = \begin{matrix} & A & B & C \\ A & 0.8 & 1/3 & 0 \\ B & 0.1 & 1/3 & 1/2 \\ C & 0.1 & 1/3 & 1/2 \end{matrix}$	**B1**		
			M1	Solve or consider Q^n for large n.	
		0.455, 0.273, 0.273 $\left(= \frac{5}{11}, \frac{3}{11}, \frac{3}{11}\right)$	**A1**		
			3		
	(v)	P(from A to A) = 0.8. So $\alpha = 0.8$	**B1**		
		$\Rightarrow \frac{\alpha}{1-\alpha} = \frac{0.8}{0.2} = 4$	**M1** **A1**	For using $\frac{\alpha}{1-\alpha}$ or $\frac{1}{1-\alpha}$	
			3		
	(vi)	New transition matrix: $\mathbf{R} = \begin{matrix} & A & B & C \\ A & 1 & 1/3 & 0 \\ B & 0 & 1/3 & 1/2 \\ C & 0 & 1/3 & 1/2 \end{matrix}$	**M1**	Method by "trial" with new matrix.	
		We need 3rd entry of 2nd row to be < 0.1 $\mathbf{R}^9 = \begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & 0.1162 \\ \dots & \dots & \dots \end{pmatrix}$, $\mathbf{R}^{10} = \begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & 0.0969 \\ \dots & \dots & \dots \end{pmatrix}$	**A1**	For sight of one value	
		So 10 days later (which is day 25).	**A1**		
			3		
	(vii)	A is the absorbing state.	**B1**		
		If it goes to A then it stays there.	**B1**	**oe**	
			2		

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