GCE

Mathematics (MEI)

Unit **4755**: Further Concepts for Advanced Mathematics

Advanced Subsidiary GCE

Mark Scheme for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

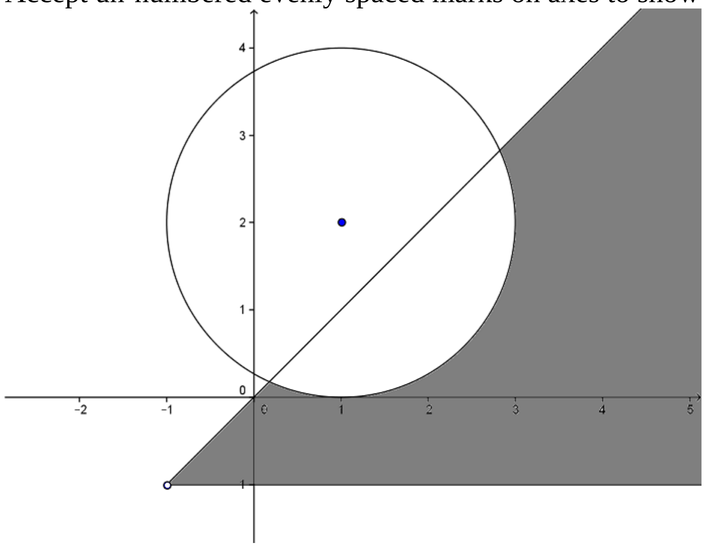
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

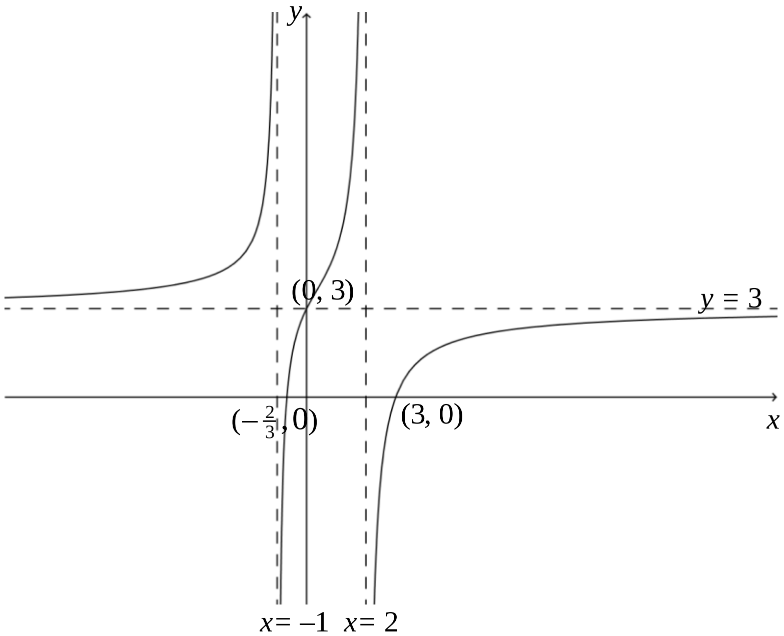
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			$\mathbf{M}^{-1} = \frac{1}{108}\begin{pmatrix} 21 & 3 \\ -8 & 4 \end{pmatrix}$	M1* M1* A1	Attempt to find $\mathbf{M}^{-1}$ or $108\mathbf{M}^{-1}$ Divide by their determinant, Δ, at some stage Correct determinant, (A0 for det **M**= $\frac{1}{108}$ stated, all other marks are available)
			$\frac{1}{108}\begin{pmatrix} 21 & 3 \\ -8 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{5}{18} \\ \frac{1}{27} \end{pmatrix}$	M1 A1	Attempt to **pre**-multiply by inverse or by Δ $\mathbf{M}^{-1}$ Correct matrix multiplication (allow one slip)
			$x = \frac{5}{18},\ y = \frac{1}{27}$,oe	A1dep* **[6]**	For both, cao x and y must be specified, may be in column vectors **SC** answers only B1
		OR			
			$4x - 3y = 1$ $8x + 21y = 3$	M1 A1	Using **M** to create two equations Correct equations
			Eliminating x or y	M1	Any valid method
			Finding second unknown	M1	Valid method
			$x = \frac{5}{18}, y = \frac{1}{27}$ Allow 3 dp or better.	A1A1	For each cao. **SC** Answers only B1
				[6]	
2			$2+3\text{j}$ and $2-3\text{j}$	B1	For both, accept $2 \pm 3\text{j}$
			Modulus $= \sqrt{(2^2+3^2)} = \sqrt{13}$	M1	Attempt at modulus of their complex roots
			Argument $= \pm\arctan\left(\frac{3}{2}\right) = \pm 0.983$	M1	Attempt at $\arctan\left(\pm\frac{3}{2}\right)$ ft their complex roots
			$2+3\text{j}$ has modulus $\sqrt{13}$ and argument 0.983 $2-3\text{j}$ has modulus $\sqrt{13}$ and argument -0.983	A1ft A1ft **[5]**	Moduli specified, ft their roots. Accept √13 only ft their roots - must be in $(-\pi, \pi]$ Accept ±0.983, ±56.3° If 2 sf given accuracy MUST be stated.

Question			Answer	Marks	Guidance
3			$\frac{-p}{2}=6 \Rightarrow p=-12$	M1,M1	M1 use of $\sum\alpha$ for p and M1 use of $\alpha\beta\gamma$ for r - allow one sign error; 2 sign errors is M1 M0
			$\frac{-r}{2}=-10 \Rightarrow r=20$	A1 A1	for p, cao for r, cao
			OR $\alpha+\beta+4=6$, $4\alpha\beta = -10$	**OR**	
			Implies α,β satisfy $2x^2-4x-5=0$	M1	Valid method to create a quadratic equation
			Roots $1\pm\frac{\sqrt{14}}{2}$	M1	Attempt to solve a 3-term quadratic
			$-\frac{p}{2}=1+\frac{\sqrt{14}}{2}+1-\frac{\sqrt{14}}{2}+4=6 \Rightarrow p=-12$	A1	for p, cao
			Product of roots $=-10=-\frac{r}{2}\Rightarrow r=20$	A1	for r, cao
			THEN	**THEN**	
			EITHER $x=4$ is a root, so $2\times 64+16p+4q+r=0$	M1	Substitution and attempt to solve for coefficient of x^2,(or for the remaining unknown.) Allow making q the subject if p and r not found.
			OR $\alpha+\beta+4=6\Rightarrow\alpha+\beta=2$ $4\alpha\beta=-10\Rightarrow\alpha\beta=-\frac{10}{4}$ $\frac{q}{2}=4\alpha+4\beta+\alpha\beta=4\times 2-\frac{5}{2}$		OR M1 using $\sum\alpha\beta$ OR use of remainder after division
			$\Rightarrow q=11$	A1 **[6]**	for q, cao

Question			Answer	Marks	Guidance
4	(i)		Accept un-numbered evenly spaced marks on axes to show scale 	B1 B1 **[2]**	Line at acute angle, all or part in Im z>0 Half line from -1- j through 0 [don't penalise if point -1- j is included] Allow near miss to 0 if $\pi/4$ marked SC correct diagram, no annotations seen B1 B0
4	(ii)			B1 B1 **[2]**	Circle centre 1 + 2j Radius 2 Must touch real axis SC correct diagram, no annotations seen B1 B0
4	(iii)			B1 B1 **[2]**	The shaded region must be outside their circle and have a border with the circumference Fully correct SC correct diagram, no annotations seen allow B1 B1
5	(i)		$\sum_{r=1}^{n}(2r-1)=2\sum_{r=1}^{n}r-n$ $=n(n+1)-n \;=\; n^2$	M1 M1 A1 **[3]**	Attempt to split into two sums (May be implied) Use of standard result for Σr cao (must be in terms of n) SC Induction: B1 case n = 1 : E1 sum to k + 1 terms correctly found : E1 argument completely correct
5	(ii)		$\dfrac{\sum_{r=1}^{n}(2r-1)}{\sum_{r=n+1}^{2n}(2r-1)}=\dfrac{n^2}{(2n)^2-n^2}$ $=\dfrac{n^2}{3n^2}=\dfrac{1}{3}=k$	M1 M1 A1 A1 **[4]**	Use of result from (i) in numerator of a fraction Expressing denominator as $\sum_{r=1}^{2n}....-\sum_{r=1}^{n}....$ need not be explicit, or other valid method. Correct sums $k=\dfrac{1}{3}$

Question			Answer	Marks	Guidance
6			$u_1 = 3$ and $\frac{3^{1-1}+5}{2} = 3$, so true for $n = 1$	B1	Must show working on given result with $n = 1$
			Assume true for $n = k$ $\Rightarrow u_k = \frac{3^{k-1}+5}{2}$	E1	Assuming true for k Allow "Let $n = k$ and (result)" "If $n = k$ and (result)" Do not allow " $n = k$" or "Let $n = k$", without the result quoted, followed by working
			$\Rightarrow u_{k+1} = 3\left(\frac{3^{k-1}+5}{2}\right) - 5$	M1	u_{k+1} with substitution of result for u_z and some working to follow
			$= \frac{3^k+15}{2} - 5$ $= \frac{3^k+15-10}{2}$		
			$= \frac{3^k+5}{2}$	A1	Correctly obtained
			$= \frac{3^{n-1}+5}{2}$ when $n = k+1$		Or target seen
			Therefore **if** true for $n = k$ it is **also true** for $n = k+1$.	E1	Both points explicit Dependent on A1 and previous E1
			Since it is true for $n = 1$, it is true for all positive integers, n.	E1 **[6]**	Dependent on B1 and previous E1
7	(i)		Asymptotes: $y = 3$, $x = 2$, $x = -1$ Crosses axes at $(0,\ 3)$ $\left(\frac{-2}{3},\ 0\right)$, $(3,\ 0)$	B1 B1 B1 B1	 (both) Allow $x = 2, -1$ Must see values for x and y if not written as co-ordinates (both) Must see values for x and y if not written as co-ordinates.
				[4]	

Question			Answer	Marks	Guidance
7	(ii)			B1 B1 B2	Intercepts labelled (single figures on axes suffice) Asymptotes correct and labelled. Allow y = 3 shown by intercept labelled at (0,3) and x = 2 and x = -1 likewise Three correct branches (-1 each error) Any poorly illustrated asymptotic approaches penalised once only.
			When x is large and positive, graph approaches y = 3 from below, e.g. for x = 100, $\dfrac{302\times97}{98\times101}=2.9...$ When x is large and negative, graph approaches y = 3 from above, e.g. for x = -100, $\dfrac{-298\times-103}{-102\times-99}=3.03...$	B1 **[5]**	Approaches to y = 3 justified There must be a result for y
7	(iii)		$y\geq3\Rightarrow0\leq x<2$ or $x<-1$	B1 B1B1 **[3]**	$x<-1$ $0\leq x<2$ (B1 for $0<x<2$ or $0\leq x\leq2$) isw any more shown

Question			Answer	Marks	Guidance	
8	(i)		$(5+4\text{j})^2 = (5+4\text{j})(5+4\text{j}) = 25+40\text{j}-16 = 9+40\text{j}$ $(5+4\text{j})^3 = -115+236\text{j}$	M1 A1 A1 **[3]**	Use of $\text{j}^2 = -1$ at least once	
8	(ii)		$\alpha^3 + q\alpha^2 + 11\alpha + r = 0$ $\Rightarrow -115+236\text{j}+9q+40q\text{j}+55+44\text{j}+r=0$ $\Rightarrow (236+40q+44)\text{j}=0$, $-115+9q+55+r=0$ $\Rightarrow q=-7$ $\Rightarrow r=123$	 M1 M1 A1ft A1ft **[4]**	 Substitute for α Compare either real or imaginary parts $q=-7$ ft their α^2 and α^3 $r=123$ ft their α^2 and α^3	
8	(iii)		$\text{f}(z) = z^3 - 7z^2 + 11z + 123$ Sum of roots = 7 $\Rightarrow (5+4\text{j})+(5-4\text{j})+w=7$ $\Rightarrow w=-3$ Roots are $5+4\text{j}$ and $5-4\text{j}$ and -3	 M1 B1 A1 **[3]**	 Valid method for the third root. (division , factor theorem, attempt at linear x quadratic with complex roots correctly used) quoted cao real root identified, A0 if extra roots found	
8	(iv)		$z\text{f}(z) = \text{f}(z) \Rightarrow (z-1)f(z)=0$ $\Rightarrow z=1$ or $\text{f}(z)=0$ $\Rightarrow z=1,\ z=-3, z=5+4\text{j}, z=5-4\text{j}$	 M1 A1ft **[2]**	 solving $z-1=0$, **and** $f(z)=0$ (may be implied) For all four solutions [ft (iii)] NB incomplete method giving z = 1 only is M0 A0	

Question			Answer	Marks	Guidance
9	(i)		$\begin{pmatrix}1 & -2\\3 & 0\end{pmatrix}\begin{pmatrix}0 & 0 & 4\\0 & 2 & 1\end{pmatrix}$	M1	Any valid method – may be implied
			$=\begin{pmatrix}0 & -4 & 2\\0 & 0 & 12\end{pmatrix}$	A1	Correct position vectors found (need not be identified)
			$A'=(0, 0),\ B'=(-4, 0),\ C'=(2, 12)$	A1ft	co-ordinates, ft their position vectors A', B', C' identifiable. Coordinates only, M1A0A1
				[3]	
9	(ii)		**M** represents a two-way stretch	B1	Stretch. (enlargement B0)
			factor 4 parallel to the *x* axis factor 2 parallel to the *y* axis	B1 B1	Directions indicated
				[3]	
9	(iii)		$\begin{pmatrix}4 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}1 & -2\\3 & 0\end{pmatrix}$	M1	Attempt at MT in correct sequence
			$=\begin{pmatrix}4 & -8\\6 & 0\end{pmatrix}$	A1	cao
			Represents the composite transformation T followed by M		
			$\begin{pmatrix}4 & -8\\6 & 0\end{pmatrix}^{-1}=\frac{1}{48}\begin{pmatrix}0 & 8\\-6 & 4\end{pmatrix}$ represents the single transformation	A1	cao
				[3]	
		OR	$\frac{1}{6}\begin{pmatrix}0 & 2\\-3 & 1\end{pmatrix}\ \ \frac{1}{8}\begin{pmatrix}2 & 0\\0 & 4\end{pmatrix}=\frac{1}{48}\begin{pmatrix}0 & 8\\-6 & 4\end{pmatrix}$	B1 M1 A1 [3]	for T^{-1} and M^{-1} correct for attempt at $T^{-1}M^{-1}$ cao
		OR	$\begin{pmatrix}0 & -16 & 8\\0 & 0 & 24\end{pmatrix}$ whence $\begin{pmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{pmatrix}\begin{pmatrix}0 & -16 & 8\\0 & 0 & 24\end{pmatrix}=\begin{pmatrix}0 & 0 & 4\\0 & 2 & 1\end{pmatrix}$	M1	Finding A'', B'' and C'' coordinates or position vectors
			$\Rightarrow\begin{pmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{pmatrix}=\frac{1}{48}\begin{pmatrix}0 & 8\\-6 & 4\end{pmatrix}$	A1 A1	For correct position vectors Inverse matrix correctly found
				[3]	

Question			Answer	Marks	Guidance
9	(iv)		Area scale factor = 48 Area of triangle ABC = 4 square units Area of triangle $A''B''C'' = 48\times$ area of triangle ABC $= 192$ (square units)	B1 M1 A1 **[3]**	Using their "48" and their area of triangle ABC, correct triangle Or other valid method cao
		OR	Finding $A''B''C''$ (0,0) (-16, 0) (8, 24) and using them Finding the area of $A''B''C''$ Area of triangle $=192$ (square units)	B1 M1 A1	$A''B''C''$ may be in (iii) Any valid method attempted cao (possibly after rounding to 3 sf)

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