OCR

Friday 16 May 2014 – Afternoon

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Use standard series formulae to find $\sum_{r=1}^{n} r(r-2)$, factorising your answer as far as possible. **[5]**

2 Fig. 2 shows the unit square, OABC, and its image, OA′B′C′, after undergoing a transformation.

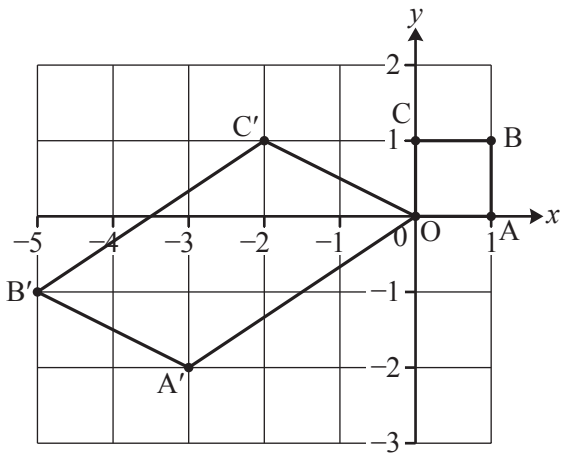

Fig. 2

(i) Write down the matrix **T** representing this transformation. **[2]**

The quadrilateral OA′B′C′ is reflected in the x-axis to give a new quadrilateral, OA″B″C″.

(ii) Write down the matrix representing reflection in the x-axis. **[1]**

(iii) Find the single matrix that will transform OABC onto OA″B″C″. **[2]**

3 You are given that $z = 2+3\mathrm{j}$ is a root of the quartic equation $z^4 - 5z^3 + 15z^2 - 5z - 26 = 0$. Find the other roots. **[7]**

4 Use the identity $\frac{1}{2r+3} - \frac{1}{2r+5} \equiv \frac{2}{(2r+3)(2r+5)}$ and the method of differences to find $\sum_{r=1}^{n} \frac{1}{(2r+3)(2r+5)}$, expressing your answer as a single fraction. **[5]**

5 The roots of the cubic equation $3x^3 - 9x^2 + x - 1 = 0$ are α, β and γ. Find the cubic equation whose roots are $3\alpha - 1$, $3\beta - 1$ and $3\gamma - 1$, expressing your answer in a form with integer coefficients. **[7]**

6 Prove by induction that $\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$. **[7]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{x^2 - 5}{(x+3)(x-2)(ax-1)}$, where a is a constant.

(i) Find the coordinates of the points where the curve crosses the x-axis and the y-axis. **[2]**

(ii) You are given that the curve has a vertical asymptote at $x = \frac{1}{2}$. Write down the value of a and the equations of the other asymptotes. **[3]**

(iii) Sketch the curve. **[4]**

(iv) Find the set of values of x for which $y > 0$. **[3]**

8 You are given the complex number $w = 2 + 2\sqrt{3}\,\mathrm{j}$.

(i) Express w in modulus-argument form. **[3]**

(ii) Indicate on an Argand diagram the set of points, z, which satisfy both of the following inequalities.

$$-\frac{\pi}{2} \leqslant \arg z \leqslant \frac{\pi}{3} \text{ and } |z| \leqslant 4$$

Mark w on your Argand diagram and find the greatest value of $|z - w|$. **[9]**

9 You are given that $\mathbf{A} = \begin{pmatrix} 1 & 3 & -1 \\ -1 & \alpha & -1 \\ -2 & -1 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3\alpha - 1 & -8 & \alpha - 3 \\ 5 & 1 & 2 \\ 2\alpha + 1 & -5 & \alpha + 3 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} \gamma & 0 & 0 \\ \beta & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix}$.

(i) Show that $\beta = 0$. **[2]**

(ii) Find γ in terms of α. **[2]**

(iii) Write down $\mathbf{A}^{-1}$ for the case when $\alpha = 2$. State the value of α for which $\mathbf{A}^{-1}$ does not exist. **[3]**

(iv) Use your answer to part **(iii)** to solve the following simultaneous equations.

$$\begin{aligned} x + 3y - z &= 25 \\ -x + 2y - z &= 11 \\ -2x - y + 3z &= -23 \end{aligned}$$ **[5]**

END OF QUESTION PAPER

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.