GCE

Mathematics

Unit **4733**: Probability and Statistics 2

Advanced GCE

Mark Scheme for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
· and ·	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (OCR) Statistics strand

a   Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.
It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c   The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. The nature of the errors allowed for the award of an M mark may often be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$\dfrac{150-\mu}{\sigma}=2.00$ $\dfrac{143-\mu}{\sigma}=-1.5$ Solve to get $\boldsymbol{\mu=146,\ \sigma=2}$	M1 A1 B1 M1 A1 A1 **6**	Standardise with σ, μ at least once, ignore cc, $\surd$ errors, equate to z Both LHS and signs of RHS correct Both z-values correct to 3 SF Correct method for solution $\mu \in [145.95, 146.05)$ www $\sigma \in [1.995, 2.005)$ or $\sigma^2 = 4$ www	z not used, e.g. equated to 0.0228 and 0.9332 or 0.5092 and 0.8246: max M0M1 One z, one not: M1A0B0 Withhold if elimination done wrongly $\surd\sigma$ or σ^2: can get M1A0B1M1A1A0 cc: M1A0B1M1A0A0
2	(i)		That they don't occur regularly or to a fixed pattern, or are unpredictable	B1 **1**	Any similar or equivalent statement, but *not* independent or equivalent Both right and wrong: B0	E.g. "no pattern": expect to be right E.g. "doesn't affect": expect to be wrong
	(ii)		Dead rabbits occur independently, i.e., one occurrence does not affect the probability of another *or* at constant <u>average</u> rate, i.e. mean number uniform along the whole road	B1 B1 **2**	Correct statement of principle Correct interpretation of that principle Context needed for any marks **SR:** "Constant <u>rate</u>" B0, correct reason can get B1 if "average" implied	*Not* "constant probability" One right, one wrong, e.g. independent + "$np <$ 5, $nq < 5$": max 1 Only "Singly" stated, implied or used: max B1 Right condition but explanation shows it's wrong: B0B0
	(iii)		Po(2.75) $e^{-2.75}\dfrac{2.75^3}{3!}=\boldsymbol{0.2215}$	M1 M1 A1 **3**	Po(1650/600) attempted Correct formula, any numerical λ Answer in range [0.221, 0.222]	Needs evidence for this <u>Must be seen</u> Formula required, so no formula $\Rightarrow$ M0A0
3	(i)		$\displaystyle\int_{-3}^{3}\dfrac{3}{2a^3}\,x^2dx=\left[\dfrac{x^3}{2a^3}\right]_{-3}^{3}=\dfrac{27}{a^3}$ $=0.125$ so $\boldsymbol{a=6}$	M1dep* B1 *M1 A1 **4**	Integrate, attempt at correct seen limits *somewhere* Correct indefinite integral, can be implied by, e.g. $27/a^3$ Equate, with limits, to 0.125 and solve Solve to get $a = 6$ exactly	Allow e.g. "< 3" = "≤ -4" Allow also for a^3 on top Allow 6.00 but no other decimals. *Not* ± 6
	(ii)		$\mu=0$ $\displaystyle\int_{-a}^{a}kx^4dx=\left[k\dfrac{x^5}{5}\right]_{-a}^{a}=\dfrac{3a^2}{5}$ $=1.35$ so $\boldsymbol{a=1.5}$	B1 M1dep* B1 *M1 A1 **5**	Stated somewhere or calculated, any a Attempt to integrate $x^2\mathrm{f}(x)$, limits $\pm a$ Or exact equivalent, can be implied Equate to 1.35 and solve $a = 1.5 \pm 0.005$, allow ± 1.5, ignore "must be positive"	If $\mu = 0$ not mentioned anywhere, or "$-\mu$" stated [instead of "$-\mu^2$"], B0 but can get remaining 4/5 Don't need explicit $-\mu^2$ here NB: $a = 3$ is *not* MR but can get B1 for $\mu = 0$
	(iii)		x is a value [values] that X takes	B1 **1**	Ignore irrelevancies or extra wrong, unless contradictory	*Not* answers just about the *function*

4	(i)	B(90, 0.05)	M1	B(90, *any*) stated or implied", but *p* can be algebraic or omitted	Not just "*n* = 90
					Exact (0.836055): M1A0M0A0B0, 1/5
		$\approx$ Po(4.5)	A1	Poisson (4.5) stated or implied	Normal (4.5, 4.275): M1A0M0A0B0, 1/5
		P($\leq$ 6) from Po(4.5)	M1	Allow M1A0 for 0.7029, 0.9134,	
		$= \mathbf{0.8311}$	A1	0.8436 or 0.8180 but nothing else	*Not* isw: final answer 0.1689 is M0A0
		n large, *p* small, *or n* > 50, *np* < 5,	B1	Either pair asserted, *n* = 90, *p* = 0.05	Allow "*np* < 5, *n* large"
		therefore Poisson	**5**		
	(ii)	B(90, 0.35) $\approx$ N(31.5, 20.475)	M1	Normal, attempt at 90×0.35	
			A1	Both parameters correct, allow $\surd$	$s^2 = 819/40$
		P($\leq$ 28) $= \Phi\left(\dfrac{28.5 - 31.5}{\sqrt{20.475}}\right)$	M1	Standardise "29" using *np*, $\sqrt{npq}$, allow wrong/no cc, σ^2 or $\sqrt{\sigma}$	Variance *np* or *nq*, or extra $\surd$90 in SD: M0 so N(31.5, 58.5) is 1/5
			A1	cc <u>and</u> $\surd$ correct	NB: Exact: 0.2558, 0/5
		$= \Phi(\text{-}0.6630) \qquad = \mathbf{0.2537}$	A1	Answer, in range [0.2535, 0.2545)	
			5		
5	(i)	Avoids (reduces) bias,	B1	– unbiased (allow "fair")	*Not:* – all equally likely to be selected
		or "representative"		– representative (allow "reliable")	– selections independent
		or "allows calculations to be done"	**1**	– allows use of distribution	– quick/easy/cheap
		or "allows reliable estimates"		Both right and wrong: B1	– random sample
	(ii)	B(18, 0.25)	M1	B(18, 0.25) stated or used	
		H$_0$: *p* = 0.25, H$_1$: *p* > 0.25	B2	One error, B1; *x* or $\bar{x}$ B0; π: B2	*Any* symbol can get B2 if explicitly defined
		α: P($\geq$ 8) = 1 – P($\leq$ 7) = **0.0569**	A1	0.0569 seen	Allow 0.9431 only if "> 0.95" and vice versa.
		> 0.05	A1	Explicit comparison with 0.05	"> 8" (0.0193), "$\leq$ 8" (0.9807) or "= 8" (.0376): max M1B2 [A0A0M0A0], 3/7
		β: CR is $\geq$ 9, and 8 < 9	A1dep*	Correct CR and explicit comparison	
		probability 0.0193	*A1	0.0193 explicitly seen	
		If more than one probability seen, assume method is β. Note that this requires explicit comparison for either A1; but can get final M1A1			
		Do not reject H$_0$.	M1	Correct first conclusion, e.g. "reject H$_1$"	M1 needs correct method, comparison, like-with-like, $\geq$ 8 (or $\leq$ 7 but only if used
		Insufficient evidence that proportion of 1's is greater than 25%.	A1ft	Interpreted, in context, consistent with *p*, acknowledge uncertainty. FT on	consistently)
			7	wrong CR/*p*	Allow "change" instead of "increase"
				Not: "significant evidence that proportion of 1s is 25%"	**SR**: 2-tail, max M1B1B0A1A0M1A1

6	(i)	$\bar{t} = 11.76$ $\hat{\sigma}^2 = \dfrac{120}{119}\left(\dfrac{18737.712}{120} - 11.76^2\right) = 18$ $H_0: \mu = 11.0,\ H_1: \mu \neq 11.0$ α: $z = \dfrac{11.76 - 11.0}{\sqrt{18/120}} = \mathbf{1.9623}$ > 1.645	B1 M1 M1 A1 B2 M1 A1 A1	11.76 seen or implied Biased estimate (= 17.85) ×120/119, *or* single formula with 119 divisor Answer 18 ± 0.05 One error, B1, but $\bar{t}$, *t*, *x* etc: B0 (*u*: B1) Standardise with 120, ignore cc or √ errors A.r.t. (±)1.96 *or* $p \in [0.0245, 0.025]$ www Compare explicitly with (±)1.645 or 0.05, consistent with their *z* or *p*. *[Needs to be "next to" TS]*	i.e. correct single formula gets M2 If both hypotheses involve 11.76, only further mark possible is next M1 [max 5/11] 120 omitted gets no further marks [max 6/11] Ignore "N(11.76, …)" *unless* hypotheses omitted altogether, in which case treat as hypotheses in terms of 11.76
		β: CV $11.0 \pm 1.645 \times \sqrt{(18/120)}$ $= 11.637$ (or 10.363) $11.76 > 11.64$	M1 A1 A1	$11.0 + z\sigma/\sqrt{120}$, needs 120 and + or ± Ignore 10.363 Explicit comparison, consistent tail	If $11.76 - z\sigma/\sqrt{120}$, give M1A0A0 M0A0 (even if correct hypotheses)
		Reject H_0. Significant evidence that the average time has changed.	M1 A1ft **11**	Correct first conclusion, allow "Accept H_1" Contextualised, acknowledge uncertainly, FT on wrong CR/*z*/*p*	Needs correct method (including 120) and comparison type, 11.0 in at least one hypothesis Allow "increase" instead of "change"
	(ii)	No, the Central Limit Theorem applies	B1 **1**	or "No, large sample". Withhold if extra wrong or irrelevant reason(s) given	Needs both "no" and reason

7	(i)	α	$H_0 : \lambda = 5$ (1 or ⅓), $H_1 : \lambda < 5$ (1 or ⅓) Po(5) $P(\le 2) = \mathbf{0.1247}$ > 0.05	B2 M1 A1 A1	One error, B1, except t, x etc: 0. Allow μ Stated or implied	$H_0: \lambda_0 = 5$, $H_1: \lambda_1 < 5$ is one error Can be implied by N(5, 5) but no more marks < 2 (0.0404) or $= 2$ (0.0843) or ≥ 2 (0.9596): no further marks. > 2 (0.8753) and compare with 0.95, OK, but must be fully consistent for more than 3 marks
		β	CR is ≤ 1 and compare 2 explicitly $p = 0.0404$	A1* dep*A1		
			If more than one probability seen, assume method is β. Note that this requires explicit comparison for either A1; but can get final M1A1			
			Do not reject H_0. Insufficient evidence of reduction in mean number of points failures	M1 A1ft **7**	Correct first conclusion, needs Po(5) Contextualised, acknowledge uncertainty, FT on wrong CR/p/comparison value	M1 needs correct method and comparison
	(ii)		Po(50) $\approx$ N(50, 50) $50 - 0.5 - 1.645 \times \sqrt{50}$ $= 37.87$ so maximum number is **37**	M1 M1 A1 M1 A1 A1 **6**	Stated or implied Normal, mean 150λ Variance/sd same $50 - z \times \sqrt{50}$, *not* $\div\sqrt{n}$, allow $\sqrt{}$ error, any cc $z = 1.645$, need $\sqrt{50}$, allow $50 + 0.5$ or 50 37 only, from cc, *not* final answer 37.86, but allow corrected to 37 after check Exact Poisson (0.0473 $\Rightarrow$ 38) is 1/6	Allow if variance $= \sqrt{50}$ etc. Ignore $50 + z \times \sqrt{50}$. $\sqrt{}$ errors here is M1A0 38(.36) from no cc is probably 5/6, but beware that exact Poisson also gives 38
8	(i)		$1 - P(\le 7)$ $= 0.0315$ or 3.15%	M1 A1 **2**	Clearly stated, or implied by any of 0.0083, 0.0933, 0.0103, 0.0576, 0.0744 Ignore subsequent "therefore 5%" etc	Other answers 0/2 unless "$1 - P(\le 7)$" explicitly seen
	(ii)		P(Type II error $\mid p = 0.5$) = 0.6047 P(Type II error $\mid p = 0.7$) = 0.0933 ⅓ × 0.6047 + ⅓ × 0.0933 [= 0.2327] P(none of 4) $= 1 - (1 - 0.2327)^4$ $= \mathbf{0.653}$	M1 A1 M1 M1 A1 **5**	Explicit "$P(\le 7)$ for $p = 0.5$" [or 0.7] or 0.2120, 0.7805: M1 Both correct ⅓ × one prob + ⅓ × other prob (*not* 0.9685) $1 - (1 - \text{ans})^4$ or equivalent binomial, not ⅓ Allow in range [0.653, 0.654]	Tail consistent. Any "$1 -$" errors: M0 Ignore 0.9685 Independent Independent [*not* $\frac{16}{81}$ or $\frac{65}{81}$] (all three M marks independent)