## GCE

## Mathematics

Unit 4730: Mechanics 3
Advanced GCE

Mark Scheme for June 2014

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. Annotations and abbreviations

| Annotation in scoris | Meaning |
| :--- | :--- |
| $\checkmark$ and $\boldsymbol{x}$ |  |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0,1 |
| B0, B1 | Independent mark awarded 0,1 |
| SC | Special case |
| $\wedge$ | Omission sign |
| MR | Misread |
| Highlighting |  |
|  |  |
| Other abbreviations  <br> in mark scheme Meaning <br> E1 Mark for explaining <br> U1 Mark for correct units <br> G1 Mark for a correct feature on a graph <br> M1 dep* Method mark dependent on a previous mark, indicated by * <br> cao Correct answer only <br> oe Or equivalent <br> rot Rounded or truncated <br> soi Seen or implied <br> www Without wrong working <br>   <br>   |  |

## 2. Subject-specific Marking Instructions for GCE Mathematics (OCR) Mechanics strand

a Annotations should be used whenever appropriate during your marking.
The $A, M$ and $B$ annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.
c The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.
E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.
d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only - differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

## When a value is given in the paper

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

## When a value is not given in the paper

Accept any answer that agrees with the correct value to 2 s.f.
ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for $g$. E marks will be lost except when results agree to the accuracy required in the question.
g Rules for replaced work
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.
h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.
'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Answer |  |  | Marks <br> M1 <br> A1 <br> A1 [3] | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | (i) | realising impulse must be in same direction as velocity, or opposite max speed $2.8\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$ min speed $1.2\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$ |  | $\begin{array}{r} 0.8+/-0.6 / 0.3 \\ -1.2 \text { is wrong } \\ \hline \end{array}$ | various methods |
|  | (ii) | Impulse momentum diagram $\cos \theta=\frac{0.6^{2}+0.24^{2}-0.75^{2}}{2 \times 0.6 \times 0.24}$ $\theta=120^{\circ}(2.098 \mathrm{rad})$ <br> angle shown correctly | M1 <br> A1 <br> M1 A1 [4] | Triangle with sides labelled $0.24,0.6$ and 0.75 or $0.8,2$ and 2.5 <br> accept $59.8^{\circ}$ ( 1.04 rad ) <br> consistent with their $\theta$; dep M1A1M1 | Allow M1 if positions wrong. <br> Diagram must be correct. <br> $v_{x}=0.8+2 \cos \theta \quad$ M1 either <br> $v_{y}=2 \sin \theta$ and correct diag A1 both <br> Square, add, giving $1.61=3.2 \cos \theta \mathrm{M} 1$ 120.(21)...A1 |
| 2 | (i) | By energy $\begin{aligned} & \frac{30(d-0.6)^{2}}{2 \times 0.6}=48 \times d \\ & 25 d^{2}-78 d+9=0 \\ & \text { or } 30 d^{2}-93.6 d+10.8=0 \\ & (d=) 3(\mathrm{~m}) \end{aligned}$ | $\begin{aligned} & \text { M1* } \\ & \text { A1 } \\ & \text { *M1 } \\ & \text { A1 [4] } \\ & \hline \end{aligned}$ | Attempt at elastic energy <br> get 3 term quadratic and attempt to solve ignore $d=0.12$, unless given as answer | Allow M1 for $\frac{30 y^{2}}{(2) \times 0.6}=k d$ $\frac{30 x^{2}}{2 \times 0.6}=48(x+0.6)$ <br> allow 1 slip or $25 x^{2}-48 x-28.8=0$ $(x=) 2.4 \text { leading to }(d=) 3$ |
|  | (ii) | Use $F=m a$ $\begin{aligned} & 48-\frac{30 \times(3-0.6-1.3)}{0.6}=( \pm) \frac{48}{g} a \\ & (a=)(+/-) 1.43 \end{aligned}$ <br> upwards | M1 <br> A1ft <br> A1 <br> A1 [4] | ft their ' 3 ' $1.4291666$ <br> depends on $a$ being right | allow missing $g$, allow 1.3 or 0.6 to be omitted <br> Using energy: $a=v \frac{\mathrm{~d} v}{\mathrm{dx}}=\frac{9}{4 \mathrm{~g}}(50 x-72) \mathrm{M} 1 \mathrm{~A} 1$ |


| Answer |  |  | MarksM1A1M1A1A1 [5] | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | (i) | Using conservation of momentum along loc $0.1 \times 2.8+0.4 \times 1 \times 0.8=0.4 \times b$ <br> Using NEL $\begin{aligned} & b-0=-e(1 \times 0.8-2.8) \\ & e=0.75 \end{aligned}$ |  | 3 (or 4) terms, correct dimensions <br> Vel diff after $=\mathrm{e} x$ vel diff before | Allow sign errors, (sin/cos) may see $b=1.5$ Allow $\pm e$ |
|  | (ii) | $\begin{aligned} & b(\text { perp })=0.6 \\ & \tan \beta=\frac{b(\text { perp })}{\text { their } 1.5}, \end{aligned}$ <br> angle turned through is $36.9^{\circ}-\beta$ $=15.1^{\circ}(0.262 \mathrm{rad})$ | $\begin{aligned} & \text { B1 } \\ & \text { M1* } \\ & \text { *M1 } \\ & \text { A1 [4] } \end{aligned}$ | $\beta=21.8^{\circ} ;$ ft 1.5 from (i) <br> Must be $36.9^{\circ}$ - their $\beta$ (soi) | May be on diagram <br> 21.8014...(0.381 rad) <br> 36.86989 <br> 15.068 scB 1 for $165^{\circ}$ after B1M1 |
| 4 | (i) | $\begin{aligned} & \text { Use } F=m v \frac{\mathrm{~d} v}{\mathrm{~d} x} \\ & -4 v=\frac{\mathrm{d} v}{\mathrm{~d} x} \\ & -4 x=\ln v+c \\ & 0=\ln 2+c \\ & \ln \frac{v}{2}=-4 x \\ & v=2 \mathrm{e}^{-4 x} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { M1 } \\ & \text { M1 } \\ & \text { A1 [5] } \end{aligned}$ | expression for $\frac{\mathrm{d} v}{\mathrm{~d} x}$ required <br> get $(+/-) A x=\ln v+c$ <br> valid attempt to find $c$ <br> need a step leading to given answer <br> AG | Allow sign error, missing $m$ or g inc |
|  | (ii) | $\begin{aligned} & \mathrm{e}^{4 x} d x=2 d t \\ & \frac{1}{4} \mathrm{e}^{4 x}=2 t+c \\ & \frac{1}{4}=0+\mathrm{c} \\ & \mathrm{e}^{4 x}=4\left(1+\frac{1}{4}\right) \\ & x=\frac{1}{4} \ln 5 \end{aligned}$ | $\begin{aligned} & \text { M1* } \\ & \text { A1 } \\ & \text { *M1 } \\ & \text { *M1 } \\ & \text { A1 [5] } \end{aligned}$ | Write v as $\frac{\mathrm{dx}}{\mathrm{dt}}$ and separate variables must have $c$ or use limits valid attempt to find $c$ or subst limits find $x$ when $t=0.5$ - need to remove $\exp$; allow even if no $c$ Accept 0.402(359...) | $\begin{aligned} & \mathrm{d} v / 4 v^{2}=-\mathrm{d} t \\ & \frac{1}{\mathrm{~d}}=4 t+\frac{1}{2} \\ & \frac{\mathrm{~d} x}{\mathrm{dt}}=\frac{2}{8 t+1} \quad \text { OR } \mathrm{t}=0.5 \text { gives } \mathrm{v}=0.4 \\ & x=\frac{1}{4} \ln (8 t+1)+c \quad \text { OR }-4 x=\ln 0.2 \\ & x=\frac{1}{4} \ln 5 \end{aligned}$ |
| 5 | (i) | Take moments about $A$ for whole body $\begin{aligned} & W \times 2 L \cos 60^{\circ}+2 W \times 6 L \cos 60^{\circ}=R \times 8 L \cos 60^{\circ} \\ & R=1.75 W \\ & S=1.25 W \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \\ & \text { B1 [4] } \end{aligned}$ | Correct 3 terms needed; dim correct $\cos 60^{\circ}$ may be omitted at least 1 correct step to show given answer | Allow sign errors, $W / 2 W, \cos /$ sin, $R$ is reaction at $C$ $S$ is reaction at $A$ For less efficient methods, M1 can only be earned when equation with one unknown, $R$, is reached. |


| Answer |  |  | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | (ii) | Take moments about $B$ for equil of $B C$ $T x L \sin 60^{\circ}+2 W x 2 L \cos 60^{\circ}=$ <br> $1.75 \mathrm{Wx} 4 L \cos 60^{\circ}$ <br> solve to get $T=\sqrt{3} W$ | $\begin{aligned} & \text { M1* } \\ & \\ & \text { A1 } \\ & * \text { M1 } \\ & \text { A1 [4] } \end{aligned}$ | Correct 3 resolved terms needed; dim correct; or for $B A$ $T \mathrm{x} L \sin 60^{\circ}+W \mathrm{x} 2 L \cos 60^{\circ}=$ $1.25 W \mathrm{x} 4 L \cos 60^{\circ}$ accept $T=1.73 W$ | allow sign errors, $W / 2 W$, cos/sin, |
|  | (iii) | Resolve vertically for $A B$ $\begin{aligned} & Y+1.25 W-W=0 \\ & Y=0.25 W, \text { downwards } \\ & X=\sqrt{3} W \text { to left } \end{aligned}$ | M1 <br> A1CAO <br> B1ft [3] | direction must be clear direction must be clear | Weight and normal term must be for same rod |
| 6 | (i) | $\begin{aligned} & \frac{1}{2} m v^{2}=m g \times 0.8\left(1-\sin 30^{\circ}\right) \\ & v=2.8 \mathrm{~m} \mathrm{~s}^{-1} \end{aligned}$ <br> Speed of P and Q equal <br> Use conservation of momentum <br> $5 m \times 2.8-m \times 2.8=5 m q+m p$ <br> Use of NEL $\begin{aligned} & p-q=-0.95(-2.8-2.8) \\ & p=6.3 \mathrm{~m} \mathrm{~s}^{-1} \\ & q=0.98 \mathrm{~m} \mathrm{~s}^{-1} \quad Q \text { moves to left } \end{aligned}$ | M1 <br> A1 <br> B1ft <br> B1ft <br> M1 <br> A1ft <br> A1 <br> A1 [8] | Or with ' $5 m$ ' if for $Q$ <br> soi <br> Ft on velocity <br> Ft on velocity supporting work required forAG direction must be clear | allow $g$ missing for M1. <br> Might see $v^{2}=0.8 g$ <br> $p$ is vel of $P, q$ is vel of $Q$, both to left Allow $\pm e$ |
|  | (ii) | By energy for $P$ at top $\begin{aligned} & \frac{1}{2} m 6.3^{2}=\frac{1}{2} m v^{2}+m g \times 1.6 \\ & v^{2}=8.33 \end{aligned}$ <br> Use $F=m a$ at top $\begin{aligned} & m g+R=m \times \frac{8.33}{0.8} \\ & R=0.6125 m \end{aligned}$ | M1 <br> A1 <br> A1 <br> M1 <br> A1ft <br> A1CAO <br> [6] | must have 3 terms <br> Soi <br> must have 3 terms their $v^{2}$ <br> Or $49 m / 80$ | allow $g$ missing, sign error allow $g$ missing, sign error |


| Answer |  |  | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | (i) | $\begin{aligned} & m g \times 0.2=\frac{2.45 m \times e}{0.3} \\ & e=0.24 \end{aligned}$ | M1 <br> A1 [2] | No errors; must show all numbers | allow sin/cos, wrong sign, missing g |
|  | (ii) | Use $F=m a$ down slope $\begin{aligned} & m g \sin \alpha-\frac{2.45 m(x-0.3)}{0.3}=m \ddot{x} \\ & \ddot{x}=-\frac{49}{6}(x-0.54) \\ & \text { SHM (about } x=0.54) \\ & \omega=7 / \sqrt{ } 6 \quad(2.8577) \\ & T=2.20 \\ & a=0.105 \mathrm{~m} \quad(0.1049795) \end{aligned}$ | M1 <br> A1 <br> A1 <br> B1 <br> B1CAO <br> B1ft [6] | 3 terms needed oe Accept 2.45/0.3 for $\omega^{2}$ <br> Dep M1A1. Must be in correct form, and $\omega^{2}$ in simplified form Soi <br> AG Need to see $2 \pi / \omega$ oe ft their $\omega \frac{3 \sqrt{6}}{70}$ | Allow sign error, $\sin /$ cos, missing $g$ or m <br> Could use $x$ in place of $x-0.3$, leading to $\frac{\vec{x}}{x}=-\frac{49}{6}(x-0.24)($ about $x=0.24)$ <br> Or $\mathrm{x}+0.24$ in place of $\mathrm{x}-0.3$ leading to $\bar{x}=-\frac{49}{6} x \quad($ about $x=0)$ <br> May see $\omega^{2}=8 \frac{1}{6}$ <br> 2.1986568... <br> NB Can find $a$ by energy, leading to $\omega$ and $T$ |
|  | (iii) | Use of SHM eqn for distance $x=-0.0956(227 \ldots)$ <br> Dist from $O$ is $0.444(377 \ldots$...) (m) Use of SHM equation for velocity $v=-0.124 \quad(-0.123949 \ldots)$ | M1 <br> A1ft <br> A1CAO <br> M1 <br> A1 [5] | $x=a \sin \omega t$ <br> Their $a$ $v=a \omega \cos \omega t$ <br> must be clear velocity is towards O | Allow M1 for $\mathrm{x}=a \cos \omega t$ Or -0.9553 or -0.09577 <br> Allow M1 for $v=-a \omega \sin \omega t$ if consistent with $x$ eqn for $\sin / \cos , a, \omega$ Use of $v^{2}=\omega^{2}\left(a^{2}-x^{2}\right)$ will not gain A1 unless direction is established |
|  |  |  |  |  |  |

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