A-LEVEL

Statistics

Statistics 4 – SS04

Mark scheme

6380

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Version/Stage: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$H_0 : p = 0.35$ $H_1 : p \neq 0.35$	B1		For both
	Under H_0 , the number buying tickets in advance ~ Bin(25, 0.35)	B1		Use of correct distribution
	Then $p(X \geq 13) = 1-0.9396$	M1		For finding $p(X \geq 13)$ or $p(X > 13)$ from a binomial distribution
	= 0.0604 which is > 0.05.	A1		(0.06(0) ~ 0.061)
	Cannot reject H_0 at the 10% level	AF1		ft their prob compared with 0.05
	No evidence of difference from 35 percent.	E1		Correct conclusion in context. Needs all previous marks.
			6	*(Note: Using p(X > 13) gives P=0.0255 then reject H_0 for max 4/6 B1B1M1A0AF1E0)*
(b)	e.g. Not a random sample/trials not independent/populations different	E1		Relevant reason
	e.g. Probably some are travelling together so likely to buy tickets the same way/maybe commuter train so many season tickets (ie 35% not relevant)/all in a carriage not comparable with all using the station.	E1		More context
			2	
		Total	**8**	

Q	Solution	Marks	Total	Comments
2(a)	No. with scratches~ B(200, 0.03)			
	Which is approx Poisson with	B1		For Poisson (stated or clearly used)
	$\lambda = 200 \times 0.03 = 6$	B1		May be implied
	Then P(X > 4) = 1 – 0.285(1)	M1		Allow for 1 – 0.151(2) (= 0.849) Allow for attempt at binomial calculation (may be implied by 0.719 or 0.853)
	= 0.715	A1		AWRT 0.715
			4	
(b)	No. with scratches (X) ~ B(10000, 0.25)	B1		
	which is approximately N(2500, 1875)	M1		Normal approximation stated or clearly used.
		A1		Mean = 2500 cao, variance = 1875 cao (or SD = 43.3 AWRT). May be implied.
	$P(X \le 2550) = P(Z < \frac{2550.5 - 2500}{\sqrt{1875}})$	M1		Standardizing (allow missing (2550) or wrong (2549.5) CC; ignore sign
	$= P(Z < 1.16(6))$	m1		Use of 2550.5 and correct sign. May be implied by Z = 1.16~1.17
	= 0.879 (from z = 1.17)	A1		AWFW 0.8765 ~ 0.879 (more exact value 0.8782)
				Notes: (i) No CC gives Z=1.15(5) & p=0.8759 for max 4/6 (ii) Wrong CC gives Z=1.14(3) & p=0.8735 for max 4/6 (iii) Use of exact B(10000, 0.25) gives answer 0.8781 which is in range but scores 1/6 if first M1 is not earned.
			6	
		Total	**10**	

Q	Solution	Marks	Total	Comments
3 (a)(i)	The 95% CI for λ using z =1.96 is	B1		For 1.96
	$58 \pm 1.96\sqrt{58}$	M1		For $c \pm z\sqrt{c}$
	$= 58 \pm 14.927$	A1		58 ± 14.9 isw or (43.1, 72.9)
			3	
(ii)	75 is outside the confidence interval (or > UCL)	AF1		ft their CI. Needs M1
	So Kelly's claim is not justified.	AFdep1		ft their CI. Needs AF1
			2	
(b)	Would not change…	E1		**Not** change Requires CI that excludes 75.
	…because the 90% CI will be narrower so still exclude 75.	Edep1		Needs (i) "narrower" or equivalent idea (eg 90% CI will be within the 95% CI OR 90% UCL < 95% UCL) OR calculate at least new UCL = 70.5 AND (ii) "exclude 75" oe. Dep on previous E1
			2	
(c)	Normal approximation (to Poisson) used			
	The standard deviation (sqrt(58)) is an estimate			
	A Poisson distribution is only assumed – may not be true.	E1,E1		Any 2 of these.
			2	
		Total	**9**	

Q	Solution	Marks	Total	Comments
4(a)(i)	Use Poisson with $\lambda = 10$.	B1		Both
	From tables, P(10) = 0.5830 – 0.4579	M1		Both terms correct. Alternatively, use correct expression $P(10) = e^{-10}\frac{10^{-10}}{10!}$
	= 0.12511	A1		AWRT 0.125
			3	
				SC Using normal approximation Use Poisson with $\lambda = 10$ (B1) $P(9.5 < X < 10.5) = P(-0.16 < Z < 0.16)$ Method and AWRT 0.16 needed (M1) = 0.56356 – (1-0.56356) = 0.12712 Gives 0.126 or 0.127 depending on rounding (A0).
(ii)	Total (T) is normal with mean 10 × 4.89= 48.9	B1		For 48.9 cao
	and variance 10×0.92^2	M1		This expression oe seen
	= 8.46(4)	A1		cao (or SD = 2.908 ~ 2.91)
	Then $P(T < 45) = P(Z < \frac{(45-48.9)}{\sqrt{Their\ variance}})$	M1		Ignore sign. Allow attempted use of CC 44.995 (or even 44.99) instead of 45. Gives subsequent answers in range.
	$= p(Z < -1.34)$ = 1- 0.90988	A1		AWRT -1.34 0.0894 ~ 0.0902
	= 0.09(012)	A1		***Equivalently**, using means:* *Using 4.5 (B1) Var = $0.92^2/10$ oe (M1) =0.0846(4) A1. Then M1A1A1 as before.*
			6	
(iii)	0.12511× (1 – 0.09012)	M1		Using their (i) × (1- their (ii)).
	= 0.11384	A1		0.113 ~ 0.114 **SC (i) × (ii)** 0.125 × 0.09 = 0.011(25) gets B1 for 0.011~0.012
			2	
(b)	$\bar{x} = 5.4286 \quad s = 1.5291$	B1		For AWRT 5.43 and s_{n-1} = 1.52 ~ 1.54 or s_n=1.4157 (1.41 ~ 1.42) (ignore labels) Or equivalent variances.
	$H_0 : \mu = 5$ $H_1 : \mu > 5$	B1		Both

	$t = \dfrac{5.4286 - 5}{1.5291/\sqrt{7}}$	M1		M1 for use of $\dfrac{s_{n-1}}{\sqrt{n}}$ *or* $\dfrac{s_n}{\sqrt{n-1}}$ in test statistic formula
		m1		Correct formula, ignore sign for m1. Or $t = \dfrac{5.4286 - 5}{1.4157/\sqrt{6}}$
	= 0.74(15)	A1		AWFW 0.74 to 0.744
	Critical value $t_6 = 1.943$	B1		For 6 df
		B1		For 1.94 cao **or** p = 0.2432 (AWFW 0.24 to 0.25)
	Accept H_0 at 5% level. No evidence that mean amount spent is over £5.	AF1		ft their t and critical t (both positive) OR ft their p-value and 0.05 but still requires positive t. Requires M1 and m1. Requires context, H_0 and H_1 the right way round, if stated, and 1-tailed test in the correct direction.
			8	***(Note:** If σ = 0.92 is used, B1B1 then nothing.)* ***Alternatively**, for full marks, may use 1-sided CI(lower limit 4.31<5,accept H_0) or 1-sided decision interval (6.12>5.43, accept H_0)*
		Total	**19**	

Mark Scheme

Q	Solution	Marks	Total	Comments
5(a)	$H_0 : p = 0.75$ $H_1 : p > 0.75$	B1		Both
	Using normal approximation to binomial	M1		Used or stated
	Using proportions, $z = \dfrac{0.78 - 0.75}{\sqrt{\dfrac{0.75 \times 0.25}{350}}} = 1.296$	M1 M1 A1		General form; allow +/-, any n. Completely correct expression; allow 0.78 for 0.75 in denominator. 1.29 ~ 1.3
	OR, using numbers $z = \dfrac{273 - 350 \times 0.75}{\sqrt{350 \times 0.75 \times 0.25}} = 1.296$	(M1) (M1) (A1)		General form as above. Completely correct expression with or without CC; allow wrong CC; 0.78 for 0.75 in denominator. 1.29 ~ 1.3 from no CC (273 used) 1.23 ~ 1.24 from correct CC (272.5 used)
	Critical value is 1.6449	B1		1.64 ~ 1.65 **Alternative for last A1B1** p=0.0975 (0.096 ~ 0.099) **or** p=0.109(0.107 ~ 0.11) from correct CC for A1, then B1 for comparison with 0.05
	Accept H_0 at the 5% level	AF1		ft their z value and normal critical value both +ve. **or** their p-value and 0.05.
	There is no evidence that the % in the UK is greater than the % for the world as a whole.	E1	8	In context, requires all previous marks.
(b)(i)	Sample proportion = $\dfrac{55}{125} = 0.44$	B1		Either
	Use of $\dfrac{(0.44)(0.56)}{125}$ (= 0.0019712)	M1		(or SD = 0.0444). Their proportion.
	95%CI: $0.44 \pm 1.96\sqrt{\dfrac{(0.44)(0.56)}{125}}$	M1		Their proportion, z and variance (M's are independent)
	Use of $z = (\pm)1.96$	B1		cao
	$= 0.44 \pm 0.087$ or (0.353, 0.527)	A1	5	Either form. Allow 0.44 ± 0.09 or (0.35, 0.53)
(ii)	Interval is entirely above 1/3 (or LCL > 0.33) It does support the claim	AF1 AFdep1		ft their CI. Needs M1 M1 ft their CI. Needs AF1 *or equivalent using numbers* *ie 41.6 < (44.1 - 65.9)*
		Total	2 **15**	

Q	Solution	Marks	Total	Comments
6 (a) (i)	$E(X) = 40 + 30 - 48 - 10 = 12$	B1		cao
	$V(X) = 4.2^2 + 2.8^2 + 2.2^2$	M1		
	$= 30.3(2)$	A1		30.3 ~ 30.32
			3	
(ii)	$P(X > 5) = P(Z > \frac{(5-12)}{\sqrt{30.32}})$	M1		Ignore sign. Their E(X) and V(X)
	$= P(Z > -1.2713)$	A1		AWRT -1.27. Needs negative
	$= 0.898$	A1		0.89 ~ 0.899
			3	
(iii)	This is the probability that Method A takes more than 5 minutes longer than Method B. (or the probability that B is more than 5 minutes shorter than A.)	E1		Any interpretation involving difference in times of the 2 methods.
		E1		Fully correct interpretation including the idea of A being "more than 5 minutes longer" than B. Thus E1 needs "probability";"more than/at least"; "5 minutes"; "A slower than B" or equivalent
			2	
(b)	Use of $C = (2/60)\times Y$ if working in minutes or $C = 2Y$ if working in hours.			
	Then $E(C) = 2/60\times 45$ or $2\times 0.75 = 1.5$	M1		Implied by correct answer
	$V(C) = (2/60)^2\times 3.3^2$ or $4\times(3.3/60)^2$	M1		Or direct use of SD(C).
	$= 0.11^2 = 0.0121$	A1		SD = 0.11cao or Var = 0.012~ 0.0121
	Alternative for first 3 marks:			
	1.5 litres used $\rightarrow 1.5/2 = 0.75$ hrs or 45 minutes mowing time	(M1) (A1)		Converting litres to time using litres/2 (0.75 and 0.875) or (45 and 52.5)
	1.75 litres used $\rightarrow 1.75/2 = 0.875$ hrs or 52.5 minutes mowing time	(M1)		Finding $P(y_1<Y< y_2)$ using consistent distribution of Y and y_1 and y_2 from sensible attempt at conversion
	$P(1.5 < C < 1.75) = P(0 < Z < 2.27(3))$	m1		Standardising at least the non-zero element using their mean and SD. Requires M1M1
	$= 0.9884 - 0.5$	m1		Difference in areas. Requires M1M1
	$= 0.488(4)$	A1		0.488~0.489
			6	
		Total	**14**	