A-LEVEL
Mathematics
Pure Core 4 – MPC4
Mark scheme

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Version 1.1: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$19x-2=A(1+6x)+B(5-x)$	M1		Correct equation and attempt to find a value for *A* or *B*.
	$A=3$	A1		
	$B=-1$	A1	3	**NMS** or cover up rule; *A* or *B* correct **SC2** *A* **and** *B* correct **SC3**.
(b)	$\int \frac{3}{5-x}-\frac{1}{1+6x}\ \mathrm{d}x$			
	$=p\ln(5-x)+q\ln(1+6x)$	M1		Condone missing brackets **OE** Either term in a correct form
	$=-3\ln(5-x)$	A1ft		ft on their *A*
	$-\tfrac{1}{6}\ln(1+6x)$	A1ft		ft on their *B*
	$\int_0^4 =\left[-3\ln 1-\tfrac{1}{6}\ln 25\right]-\left[-3\ln 5-\tfrac{1}{6}\ln 1\right]$	m1		Substitute limits correctly in their integral; $\mathrm{F}(4)-\mathrm{F}(0)$
	$=-\tfrac{1}{6}\ln 25+3\ln 5$	A1		ACF. $\ln 1=0$ PI
	$=\frac{8}{3}\ln 5$	A1	6	CSO Condone equivalent fractions or recurring decimal
	Total		**9**	

Q2	Solution	Mark	Total	Comment
(a)	$R=\sqrt{29}$	B1		Allow 5.4 or better
	$\sqrt{29}\cos\alpha=2, \sqrt{29}\sin\alpha=5$ or $\tan\alpha=\tfrac{5}{2}$	M1		Their $\sqrt{29}$ Note $\cos\alpha=2$ or $\sin\alpha=5$ is M0
	$\alpha=1.19$	A1	3	Must be exactly this
(b)(i)	$R\cos(x+\alpha)=R$ or $\cos(x+\alpha)=1$ or $x+\alpha=2\pi$ or $x+\alpha=0$ or $x=-\alpha$	M1		Candidate's R and α
	$(x=)$ 5.09	A1	2	Must be exactly this
(ii)	$\cos(x+\alpha)=-\frac{1}{R}$	M1		Candidate's R and α; PI
	$(x+\alpha=)$ 1.75757... and 4.52560...	A1		Rounded or truncated to at least 2 dp; Ignore 'extra' solutions
	$x=0.567$ and $x=3.34$	A1	3	Condone $x=0.568$; $x=3.34$ must be correct NMS is 0/3 **A0** if extra values in interval $0<x<2\pi$
	Total		**8**	

Q3	Solution	Mark	Total	Comment
(a)	$\mathrm{f}\left(-\frac{1}{2}\right) = -1-3+1+d = -2$	M1		Attempt to evaluate $\mathrm{f}\left(-\frac{1}{2}\right)$ **and** equated to -2
	$d = 1$	A1	2	NMS is 0/2
(b)(i)	$(2x+1)$ is a factor	B1		**OE** $\left(x+\frac{1}{2}\right)$
	$\mathrm{g}(x) = (2x+1)\left(4x^2+bx+3\right)$	M1		Attempt to find quadratic factor or a second linear factor using Factor Theorem
	$\mathrm{g}(x) = (2x+1)\left(4x^2-8x+3\right)$			**OE** if $\left(x+\frac{1}{2}\right)$ is used
	$\mathrm{g}(x) = (2x+1)(2x-1)(2x-3)$			**OE**; must be a product
		A1	3	**NMS** : **SC3** if product is correct **SC1** if one or two factors are correct
(ii)	$\frac{4x^2-1}{\mathrm{g}(x)} = \frac{1}{2x-3}$	B1		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2x-3}\right) = \frac{k}{(2x-3)^2}$	M1		Attempt to differentiate simplified h
	$= -\frac{2}{(2x-3)^2}$	A1		Correct derivative
	(Derivative is) negative, or < 0 hence decreasing	E1	4	Explanation and conclusion required Derivative must be correct
	Total		**9**	
(b)(ii)	**Special case** $\mathrm{h}(x) = \frac{1}{2x-3}$	B1		
	$2x-3$ is an increasing function, so $\frac{1}{2x-3}$ is a decreasing function	E1	2	Award only if $\mathrm{h}(x) = \frac{1}{2x-3}$ is correct

Q4	Solution	Mark	Total	Comment
(a)	$1+\frac{1}{5}\times 5x+kx^2$	M1		k any non-zero numerical expression
	$1+x-2x^2$	A1	2	Simplified to this
(b) (i)	$(8+3x)^{-\frac{2}{3}}=8^{-\frac{2}{3}}\left(1+\frac{3}{8}x\right)^{-\frac{2}{3}}$	B1		ACF for $8^{-\frac{2}{3}}=\frac{1}{4}$
	$\left(1+\frac{3}{8}x\right)^{-\frac{2}{3}}$			
	$=1+\left(-\frac{2}{3}\right)\left(\frac{3}{8}x\right)+\frac{1}{2}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(\frac{3}{8}x\right)^2$	M1		Expand correctly using their $\frac{3}{8}x$ Condone poor use of or missing brackets
	$\frac{1}{4}-\frac{1}{16}x+\frac{5}{256}x^2$	A1	3	Accept $=\frac{1}{4}\left(1-\frac{1}{4}x+\frac{5}{64}x^2\right)$
(ii)	$x=\frac{1}{3}$	M1		$x=\frac{1}{3}$ **used** in their expansion from (b)(i)
	0.2313 (4dp)	A1	2	Note **3** in **4th** decimal place
	Total		**7**	

Q5	Solution	Mark	Total	Comment
(a)	$\left(\frac{dx}{dt}=\right)-2\sin 2t \qquad \left(\frac{dy}{dt}=\right)\cos t$	B1		Both correct
	$\left(\frac{dy}{dx}=\right)\frac{\cos t}{-2\sin 2t}$	M1		Correct use of chain rule with their derivatives of form $a\sin 2t,\ b\cos t$
	At $t=\frac{\pi}{6}$ gradient $m_T=-\frac{1}{2}$	A1	3	
(b)	Gradient of normal $m_N=2$	B1ft		ft gradient of tangent; $m_N=\frac{-1}{m_T}$
	$\left(y-\cos\left(\frac{2\pi}{6}\right)\right)=m_N\left(x-\sin\left(\frac{\pi}{6}\right)\right)$	M1		For m_N, allow their m_T with a change of sign or the reciprocal at $\left(\sin\frac{\pi}{6},\cos\frac{2\pi}{6}\right)$ or $\left(\frac{1}{2},\frac{1}{2}\right)$
	$y=2x-\frac{1}{2}$	A1	3	Must be in this $y=mx+c$ form
	Alternative for M1 $\sin\left(\frac{\pi}{6}\right)=2\cos\left(\frac{2\pi}{6}\right)+c$			Use $y=mx+c$ to find c with their gradient m_N at $\left(\sin\frac{\pi}{6},\cos\frac{2\pi}{6}\right)$ or $\left(\frac{1}{2},\frac{1}{2}\right)$
(c)	$\cos 2q=1-2\sin^2 q$	B1		Seen or used in this form
	$\sin q=2\left(1-2\sin^2 q\right)-\frac{1}{2}$	M1		Use parametric equations and candidate's $\cos 2q$ in the form $\pm 1+k\sin^2 q$
	$8\sin^2 q+2\sin q-3=0$ **OE**	A1		Collect like terms; must be a quadratic equation
	$\left(\sin q=\frac{1}{2}\right) \qquad \sin q=-\frac{3}{4}$	A1		Must come from a correct quadratic equation with the previous 3 marks awarded
	$(x=)\ -\frac{1}{8}$	A1	5	Previous 4 marks must have been awarded
	Total		**11**	

Mark scheme Alternative

Q5	Solution	Mark	Total	Comment
(a)	$x=1-2y^2 \quad 1=-4y\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}=-4y$	B1		Find a correct Cartesian equation and differentiate implicitly correctly
	$\dfrac{dy}{dx}=-\dfrac{1}{4\sin\frac{\pi}{6}}$	M1		Use $y=\sin\frac{\pi}{6}$ or $y=\frac{1}{2}$ in their $\frac{dy}{dx}$; PI
	At $t=\dfrac{\pi}{6}$ gradient $m_T=-\dfrac{1}{2}$	A1	3	CSO
(b)	Gradient of normal $=2$	B1ft		ft gradient of tangent, $m_N=\frac{-1}{m_T}$
	$\left(y-\cos\left(\dfrac{2\pi}{6}\right)\right)=m_N\left(x-\sin\left(\dfrac{\pi}{6}\right)\right)$	M1		For m_N, allow their m_T with a change of sign or the reciprocal at $\left(\sin\frac{\pi}{6},\cos\frac{2\pi}{6}\right)$ or $\left(\frac{1}{2},\frac{1}{2}\right)$
	$y=2x-\dfrac{1}{2}$	A1	**3**	CSO
	Alternative for M1 $\sin\left(\dfrac{\pi}{6}\right)=2\cos\left(\dfrac{2\pi}{6}\right)+c$			Use $y=mx+c$ to find c with candidate's gradient m_N at $\left(\sin\frac{\pi}{6},\cos\frac{2\pi}{6}\right)$ or $\left(\frac{1}{2},\frac{1}{2}\right)$
(c)	$x=1-2y^2$	B1		PI by $x=1-2\left(2x-\frac{1}{2}\right)^2$
	$1-2y^2=\dfrac{y+\frac{1}{2}}{2}$	M1		Use their Cartesian equation and normal to eliminate x
	$4y^2+y-\frac{3}{2}=0\Rightarrow$ $8\sin^2 q+2\sin q-3=0$	A1		Collect like terms; must be a quadratic equation
	$\left(\sin q=\dfrac{1}{2}\right) \quad \sin q=-\dfrac{3}{4}$	A1		Must come from a correct quadratic equation with the previous 3 marks awarded
	$(x=)\ -\dfrac{1}{8}$	A1	**5**	Previous 4 marks must have been awarded
	Total		**11**	

Q6	Solution	Mark	Total	Comment
(a)	$\left(\overrightarrow{AB}=\right)\begin{bmatrix}2\\-4\\-6\end{bmatrix}$	B1		Or $\left(\overrightarrow{BA}=\right)\begin{bmatrix}-2\\4\\6\end{bmatrix}$
	$\overrightarrow{AB}\bullet\begin{bmatrix}3\\1\\-2\end{bmatrix}=(2\times3)+(-4\times1)+(6\times-2)$	M1		Correctly ft on "their" $\overrightarrow{AB}$
	$\sqrt{56}\sqrt{14}\cos BAC=14$	m1		Correct use of formula with consistent vectors; ACF
	angle $BAC=60°$	A1	4	or $\pi/3$; **NMS** $60°$ scores **0/4**
(b)	$\left(\overrightarrow{BC}=\right)\begin{bmatrix}3\\2\\10\end{bmatrix}+\lambda\begin{bmatrix}3\\1\\-2\end{bmatrix}-\begin{bmatrix}5\\-2\\4\end{bmatrix}$	B1		$\pm\overrightarrow{BC}$ **ACF**
	$\overrightarrow{AB}\bullet\overrightarrow{BC}=$ $2(3\lambda-2)-4(\lambda+4)-6(-2\lambda+6)=0$	M1		Correct scalar product with their $\overrightarrow{AB}$, their $\overrightarrow{BC}$, equate to 0 and solve for λ
	$14\lambda-56=0 \Rightarrow \lambda=4$	A1		
	C is at $(15,6,2)$	A1	4	Accept as a column vector **NMS** (15,6,2) scores **0/4**
(c)	E_1 is at $(11,0,0)$	B1		Accept as a column vector
	$\overrightarrow{OD}=\overrightarrow{OC}+\overrightarrow{AB}=\begin{bmatrix}15\\6\\2\end{bmatrix}+\begin{bmatrix}2\\-4\\-6\end{bmatrix}=\begin{bmatrix}17\\2\\-4\end{bmatrix}$	B1		
	$\overrightarrow{OE_2}=\begin{bmatrix}17\\2\\-4\end{bmatrix}+\frac{1}{2}\times4\begin{bmatrix}3\\1\\-2\end{bmatrix}$	M1		Correct vector expression with their λ and their $\overrightarrow{OD}$
	E_2 is at $(23,4,-8)$	A1	4	Accept as a column vector
	Total		**12**	
(b)	**Alternative by Pythagoras**			
	$\left(\overrightarrow{BC}=\right)\begin{bmatrix}3\\2\\10\end{bmatrix}+\lambda\begin{bmatrix}3\\1\\-2\end{bmatrix}-\begin{bmatrix}5\\-2\\4\end{bmatrix}$	B1		$\pm\overrightarrow{BC}$ **ACF**
	$(3\lambda)^2+(\lambda)^2+(-2\lambda)^2$ $=56+(-2+3\lambda)^2+(4+\lambda)^2+(6-2\lambda)^2$	M1		$AC^2=AB^2+BC^2$ Correct Pythagoras expression, attempt to expand and solve for λ
	$112-28\lambda=0 \quad \lambda=4$	A1		
	C is at $(15,6,2)$	A1	4	Accept as a column vector

(b)	**Alternative by** $\cos 60 = \frac{1}{2}$			
	$\frac{1}{2} = \frac{\lvert\overrightarrow{AB}\rvert}{\lvert\overrightarrow{AC}\rvert} = \frac{\sqrt{56}}{\sqrt{(3\lambda)^2 + (\lambda)^2 + (-2\lambda)^2}}$	**B1**		
	$\frac{1}{4} = \frac{56}{14\lambda^2}$	**M1**		Square and simplify
	$\lambda^2 = 16 \Rightarrow \lambda = 4 \quad (\text{or } \lambda = -4)$	**A1**		
	C is at $(15, 6, 2)$	**A1**	**4**	Accept as a column vector

(c)	**Alternatives**			
Alt (i)				
	$\overrightarrow{OE_1} = \overrightarrow{OB} + \frac{1}{2}\overrightarrow{AC} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$			
	E_1 is at $(11, 0, 0)$	**B1**		
	$\overrightarrow{OE_2} = \overrightarrow{OB} + 3\overrightarrow{BE_1} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + 3\begin{bmatrix} 6 \\ 2 \\ -4 \end{bmatrix}$	**M1**		Correct vector expression with their $\overrightarrow{BE_1}$
		B1`		All correct
	E_2 is at $(23, 4, -8)$	**A1**	**4**	
Alt (ii)				
	$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{AC} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + \begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$			
	D is at $(17, 2, -4)$	**B1**		
	$\overrightarrow{OE_2} = \overrightarrow{OD} + \frac{1}{2}\overrightarrow{AC} = \begin{bmatrix} 17 \\ 2 \\ -4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$	**M1**		Correct vector expression with their $\overrightarrow{OD}$ and their $\overrightarrow{AC}$
	E_2 is at $(23, 4, -8)$	**A1**		
	$\overrightarrow{OE_1} = \overrightarrow{OB} + \frac{1}{2}\overrightarrow{AC} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$			
	E_1 is at $(11, 0, 0)$	**B1**	**4**	

Q7	Solution	Mark	Total	Comment
(a)	$k=\left(\frac{1}{2}\right)^3+2\mathrm{e}^{-3\ln 2}\times\frac{1}{2}-\ln 2$ $=\frac{1}{8}+\frac{1}{8}-\ln 2=\frac{1}{4}-\ln 2$	B1	1	Clear use of $\left(\frac{1}{2}\right)^3=\frac{1}{8}$ and $\mathrm{e}^{-3\ln 2}=\frac{1}{8}$ Accept $\frac{2}{8}-\ln 2$
(b)	$3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	B1		
	$py\mathrm{e}^{-3x}+q\mathrm{e}^{-3x}\frac{\mathrm{d}y}{\mathrm{d}x}$	M1		
	$-6y\mathrm{e}^{-3x}+2\mathrm{e}^{-3x}\frac{\mathrm{d}y}{\mathrm{d}x}$	A1		
	$-1=0$	B1		Both required -1 and no other terms
	$\frac{3}{4}\frac{\mathrm{d}y}{\mathrm{d}x}-6\times\frac{1}{8}\times\frac{1}{2}+2\times\frac{1}{8}\frac{\mathrm{d}y}{\mathrm{d}x}-1\quad(=0)$	m1		Substitute $x=\ln 2$ or $\mathrm{e}^{-3x}=\frac{1}{8}$ and $y=\frac{1}{2}$ into their expression
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{11}{8}$ or 1.375	A1	6	
	Total		7	

Q8	Solution	Mark	Total	Comment
(a)(i)	$\int \frac{1}{\sqrt{4+5x}}\,\mathrm{d}x = \int \frac{1}{5(1+t)^2}\,\mathrm{d}t$	**B1**		Correct separation and notation seen on a single line somewhere in their solution
	$a(4+5x)^{\frac{1}{2}}$ or $b(1+t)^{-1}$	**M1**		OE $a\sqrt{4+5x}$ or $b\left(\frac{1}{1+t}\right)$
	$\frac{2}{5}(4+5x)^{\frac{1}{2}}$	**A1**		OE $\frac{2}{5}\sqrt{4+5x}$
	$-\frac{1}{5}(1+t)^{-1}\quad(+C)$	**A1**		OE $-\frac{1}{5(1+t)}$
	$x=0,\ t=0 \quad\Rightarrow\quad C=1$	**m1**		Use $(0,0)$ to find a constant
	$\frac{2}{5}(4+5x)^{\frac{1}{2}} = 1-\frac{1}{5}(1+t)^{-1}$	**A1**		**OE**
	$x=\frac{5}{4}\left(1-\frac{(1+t)^{-1}}{5}\right)^2-\frac{4}{5}$	**A1**	7	ACF eg $x=\frac{1}{20}\left(\frac{4+5t}{1+t}\right)^2-\frac{4}{5}$
(b)(i)	$\frac{\mathrm{d}r}{\mathrm{d}t}$	**B1**		Seen; allow R for r
	$\frac{1}{r^2}$	**M1**		$\frac{1}{r^2}$ seen **;** allow R for r
	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{k}{r^2}$	**A1**	3	Any constant k including $\frac{c}{\pi}$ but **not** including variable t Must use R or r consistently
(ii)	$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)=4.5=\frac{k}{1^2}$ or $4.5=\frac{c}{\pi\times 1^2}$	**M1**		**Use** $\frac{\mathrm{d}r}{\mathrm{d}t}=4.5$ with $r=1$ to find a value for the constant
	$0.5=\frac{4.5}{r^2}\Rightarrow r=3$ (metres)	**A1**	2	
	Total		**12**	