Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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8	
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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MM2B

Unit Mechanics 2B

Monday 22 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A particle, of mass $4\,\text{kg}$, moves in a horizontal plane under the action of a single force, $\mathbf{F}$ newtons. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the horizontal plane, perpendicular to each other.

At time t seconds, the velocity of the particle, $\mathbf{v}\,\text{m s}^{-1}$, is given by

$$\mathbf{v} = 4\cos 2t\ \mathbf{i} + 3\sin t\ \mathbf{j}$$

(a) (i) Find an expression for the force, $\mathbf{F}$, acting on the particle at time t seconds.

[3 marks]

(ii) Find the magnitude of $\mathbf{F}$ when $t = \pi$.

[2 marks]

(b) When $t = 0$, the particle is at the point with position vector $(2\mathbf{i} - 14\mathbf{j})$ metres. Find the position vector, $\mathbf{r}$ metres, of the particle at time t seconds.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

2 A uniform rod AB, of mass $4\,\text{kg}$ and length 6 metres, has three masses attached to it. A $3\,\text{kg}$ mass is attached at the end A and a $5\,\text{kg}$ mass is attached at the end B. An $8\,\text{kg}$ mass is attached at a point C on the rod.

Find the distance AC if the centre of mass of the system is $4.3\,\text{m}$ from point A.

[4 marks]

QUESTION PART REFERENCE

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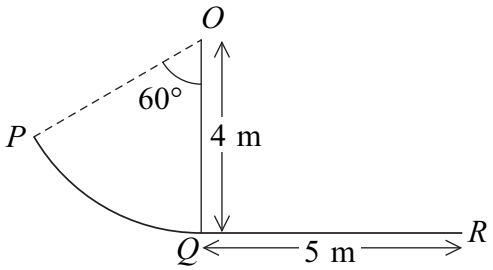
3 A diagram shows a children's slide, PQR.



Simon, a child of mass $32\,\text{kg}$, uses the slide, starting from rest at P. The curved section of the slide, PQ, is one sixth of a circle of radius 4 metres so that the child is travelling horizontally at point Q. The centre of this circle is at point O, which is vertically above point Q. The section QR is horizontal and of length 5 metres.

Assume that air resistance may be ignored.

(a) Assume that the two sections of the slide, PQ and QR, are both smooth.

(i) Find the kinetic energy of Simon when he reaches the point R.

[3 marks]

(ii) Hence find the speed of Simon when he reaches the point R.

[2 marks]

(b) In fact, the section QR is rough.

Assume that the section PQ is smooth.

Find the coefficient of friction between Simon and the section QR if Simon comes to rest at the point R.

[4 marks]

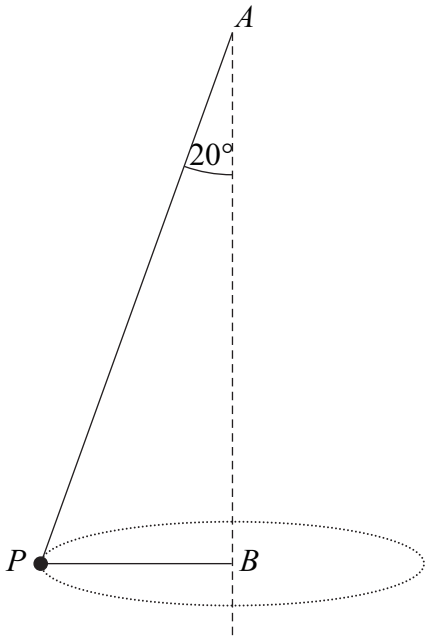
QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

4 A particle, P, of mass $5\,\text{kg}$ is attached to two light inextensible strings, AP and BP. The other ends of the strings are attached to the fixed points A and B. The point A is vertically above the point B. The particle moves at a constant speed, $v\,\text{m s}^{-1}$, in a horizontal circle of radius 0.6 metres with centre B. The string AP is inclined at $20°$ to the vertical, as shown in the diagram. Both strings are taut when the particle is moving.



(a) Find the tension in the string AP.

[3 marks]

(b) The speed of the particle is $v\,\text{m s}^{-1}$.

Show that the tension, T_{BP}, in the string BP is given by

$$T_{BP} = \frac{25}{3}v^2 - 5g\tan 20°$$

[3 marks]

(c) Find v when the tensions in the two strings are equal.

[4 marks]

QUESTION PART REFERENCE

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Answer space for question 4

5 An item of clothing is placed inside a washing machine. The drum of the washing machine has radius $30\,\text{cm}$ and rotates, about a fixed horizontal axis, at a constant angular speed of 900 revolutions per minute.

Model the item of clothing as a particle of mass $0.8\,\text{kg}$ and assume that the clothing travels in a vertical circle with constant angular speed.

Find the minimum magnitude of the normal reaction force exerted by the drum on the clothing and find the maximum magnitude of the normal reaction force exerted by the drum on the clothing.

[6 marks]

QUESTION PART REFERENCE

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Answer space for question 5

6 A van, of mass $1400\,\text{kg}$, is accelerating at a constant rate of $0.2\,\text{m s}^{-2}$ as it travels up a slope inclined at an angle θ to the horizontal.

The van experiences total resistance forces of $4000\ \text{N}$.

When the van is travelling at a speed of $20\,\text{m s}^{-1}$, the power output of the van's engine is $91.1\,\text{kW}$.

Find θ.

[9 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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7 A parachutist, of mass $72\,\text{kg}$, is falling vertically. He opens his parachute at time $t = 0$ when his speed is $30\,\text{m s}^{-1}$. He then experiences an air resistance force of magnitude $240v$ newtons, where $v\,\text{m s}^{-1}$ is his speed at time t seconds.

(a) When $t > 0$, show that $-\frac{3}{10}\frac{\text{d}v}{\text{d}t} = v - 2.94$.

[2 marks]

(b) Find v in terms of t.

[5 marks]

(c) Sketch a graph to show how, for $t \geqslant 0$, the parachutist's speed varies with time.

[2 marks]

QUESTION PART REFERENCE

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8 Carol, a bungee jumper of mass $70\,\text{kg}$, is attached to one end of a light elastic cord of natural length 26 metres and modulus of elasticity $1456\ \text{N}$. The other end of the cord is attached to a fixed horizontal platform which is at a height of 69 metres above the ground.

Carol steps off the platform at the point where the cord is attached and falls vertically. Hooke's law can be assumed to apply whilst the cord is taut.

Model Carol as a particle and assume air resistance to be negligible.

When Carol has fallen x m, her speed is $v\,\text{m s}^{-1}$.

(a) By considering energy, show that

$$5v^2 = 306x - 4x^2 - 2704 \quad \text{for} \quad x \geqslant 26$$

[4 marks]

(b) Why is the expression found in part **(a)** not true when x takes values less than 26?

[1 mark]

(c) Find the maximum value of x.

[2 marks]

(d) (i) Find the distance fallen by Carol when her speed is a maximum.

[2 marks]

(ii) Hence find Carol's maximum speed.

[1 mark]

QUESTION PART REFERENCE

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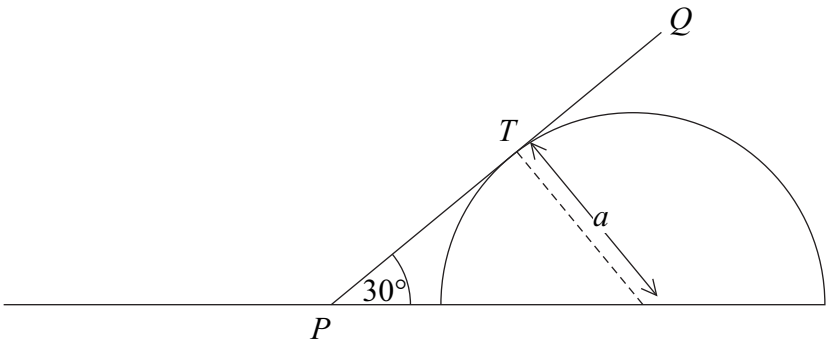
9 A uniform rod, PQ, of length $2a$, rests with one end, P, on rough horizontal ground and a point T resting on a rough fixed prism of semicircular cross-section of radius a, as shown in the diagram. The rod is in a vertical plane which is parallel to the prism's cross-section. The coefficient of friction at both P and T is μ.



The rod is on the point of slipping when it is inclined at an angle of $30°$ to the horizontal.

Find the value of μ.

[8 marks]

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END OF QUESTIONS

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