A-LEVEL

Mathematics

Further Pure 2 – MFP2

Mark scheme

6360

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Version/Stage: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

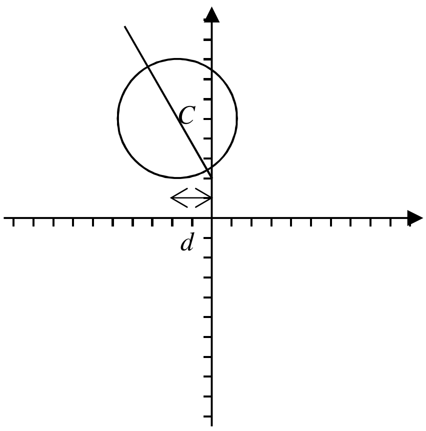
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1 (a)	$r=9$	B1		
	$\theta=-\frac{\pi}{2}$	B1		condone $-1.57\ldots$ here only
			2	$-9\mathrm{i}=9\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}$
(b)	$r=\sqrt{3}$	B1√		follow through $(\textit{their } r)^{\frac{1}{4}}$; accept $9^{\frac{1}{4}}$ etc
	$(\textit{their }\theta)/4$	M1		generous
	$\theta=-\frac{5\pi}{8},-\frac{\pi}{8},\frac{3\pi}{8},\frac{7\pi}{8}$	A1		two angles correct in correct interval
		A1		**exactly four** angles correct mod 2π
	$\sqrt{3}\,\mathrm{e}^{-\frac{\mathrm{i}5\pi}{8}},\sqrt{3}\,\mathrm{e}^{-\frac{\mathrm{i}\pi}{8}},\sqrt{3}\,\mathrm{e}^{\frac{\mathrm{i}3\pi}{8}},\sqrt{3}\,\mathrm{e}^{\frac{\mathrm{i}7\pi}{8}}$	A1	5	four correct roots in correct interval and in given form; accept $3^{\frac{1}{2}}$ for $\sqrt{3}$
	Total		7	

1(a) Accept correct values of r and θ for full marks without candidates actually writing $9\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}$.
Do not accept angles outside the required interval.
Example "$\theta=-\frac{\pi}{2}$ or $\theta=\frac{3\pi}{2}$" scores **B0**

(b) Condone $r=1.73\ldots$ for **B1** only. Do not follow through a negative value of r for **B1√**.
Example $\theta=\frac{3\pi}{8},\frac{7\pi}{8},\frac{11\pi}{8},\frac{15\pi}{8}$ scores **M1 A1 A1**
Example $\sqrt{3}\,\mathrm{e}^{-\frac{\mathrm{i}\pi}{8}+\frac{\mathrm{i}k\pi}{2}}$ scores **B1 M1** then $k=-1, 0, 1, 2$ scores **A1 A1** with final **A1** only earned when four roots are written in given form

Q	Solution	Mark	Total	Comment
2(a)	Straight line	M1		
	Half line from 2 on Im axis	A1		not vertical or horizontal
	Making approx. 30° to positive Im axis & 60° to negative Re axis	A1	3	
(b)(i)	Circle with centre on 'their' L	M1		
	Circle correct and touching Im $z = 2$	A1	2	lowest point of circle at approx 2
				
(b)(ii)	$d = 3\tan\frac{\pi}{6}$	M1		any correct expression for distance or $\frac{b-2}{a} = -\sqrt{3}$ for **M1**
	$a = -\sqrt{3}$	A1		condone -1.73 or better
	$b = 5$	B1	3	centre is $-\sqrt{3}+5\text{i}$
	Total		**8**	

(a)	The two **A1** marks are independent.
(b) (i)	If candidate draws a horizontal line at Im $z = 2$ then award **A1** if there is a clear intention for their circle to touch this line. Allow freehand circle where centre is intended to be on "their" L for **M1** but withhold **A1** if L is in wrong quadrant or drawing of circle is very poor. Award **A0** if candidate has not scored full marks in **(a)** .

Q	Solution	Mark	Total	Comment
3 (a)	$k^2+7k+14$	B1	1	
(b)	When $n=1$ LHS = $1\times2\times1=2$ RHS = 16 – 14 = 2 Therefore true for $n=1$	B1		
	Assume formula is true for $n=k$ (*) Add (k+1)th term (to both sides)	M1		(k+1)th term must be correct
	$\sum_{r=1}^{k+1} r(r+1)\left(\frac{1}{2}\right)^{r-1}$ $=16-\left(k^2+5k+8\right)\left(\frac{1}{2}\right)^{k-1}$ $+(k+1)(k+2)\left(\frac{1}{2}\right)^{k}$	A1		**A0** if only considering RHS
	$=16-\left(\frac{1}{2}\right)^{k}\left(2k^2+10k+16-k^2-3k-2\right)$ $=16-\left(\frac{1}{2}\right)^{k}\left(k^2+7k+14\right)$	A1		
	$=16-\left(\left(k+1\right)^2+5\left(k+1\right)+8\right)\left(\frac{1}{2}\right)^{k}$	A1		from part (a)
	Hence formula is true when $n=k+1$ (**) but true for $n=1$ so true for $n=2, 3, \ldots$ by induction (***)	E1	6	must have (*), (**) and (***) and must have earned previous 5 marks
	Total		**7**	
(b)	For **B1,** accept "n=1 RHS=LHS=2" but must mention here or later that the result is "true when n=1" Alternative to (***) is "therefore true for all positive integers n" etc However, " true for all $n\geqslant1$ " is incorrect and scores **E0** May define P(k) as the "proposition that the formula is true when $n=k$ " and earn full marks. However, if P(k) is not defined then allow **B1** for showing P(1) is true but withhold **E1** mark.			

Q	Solution	Mark	Total	Comment
4 (a) (i)	$\alpha+\beta+\gamma=-2$ $\alpha\beta+\beta\gamma+\gamma\alpha=3$	B1 B1	 2	
(ii)	$\alpha^2+\beta^2+\gamma^2$ $=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$ $=4-6=-2$	 M1 A1cso	 2	 correct formula **AG** be convinced; must see $4-6$ **A0** if $\alpha+\beta+\gamma$ or $\alpha\beta+\beta\gamma+\gamma\alpha$ not correct
(b) (i)	$\sum(\alpha+\beta)(\beta+\gamma)=\sum\alpha^2+3\sum\alpha\beta$ $=-2+9$ $=7$	M1 m1 A1	 3	or may use $12+4\sum\alpha+\sum\alpha\beta$ ft their $\alpha\beta+\beta\gamma+\gamma\alpha$
(ii)	$\alpha\beta\gamma=4$ $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$ $=\sum\alpha\sum\alpha\beta-\alpha\beta\gamma$ $=-6-4$ $=-10$	B1 M1 m1 A1	 4	**PI** when earning **m1** later or $(-2-\alpha)(-2-\beta)(-2-\gamma)$ $=-8-4\sum\alpha-2\sum\alpha\beta-\alpha\beta\gamma$ Sub their $\sum\alpha,\ \sum\alpha\beta\ \&\ \alpha\beta\gamma$
(c)	Sum of new roots $=2\sum\alpha=-4$ $z^3\pm4z^2+\text{"their 7"}z-\text{"their }-10\text{"}\ (=0)$ New equation $z^3+4z^2+7z+10=0$	B1 M1 A1	 3	***or*** **NMS** coefficient of z^2 written as +4 correct sub of their results from part **(b)** **Alternative** $y=-2-z$ **B1** $(-2-y)^3+2(-2-y)^2+3(-2-y)-4=0$ **M1** $y^3+4y^2+7y+10=0$ **A1** **NB** candidate may do this first and then obtain results for part (b)
	Total		**14**	

(a)(ii)	Accept $\left(\sum\alpha\right)^2=\sum\alpha^2+2\sum\alpha\beta$ etc for **M1**
(b)(ii)	If **B1** not earned, award **m1** for using $\alpha\beta\gamma=\pm4$ **.**
(c)	For **M1** the signs of coefficients must be correct FT their results from **(b)** but condone missing "= 0" However, for **A1** the equation must be correct (any variable) including "= 0"

Q	Solution	Mark	Total	Comment
5(a)	$\left(e^{\theta}-e^{-\theta}\right)^3 = e^{3\theta}-3e^{\theta}+3e^{-\theta}-e^{-3\theta}$ OE	B1		correct expansion; terms need not be combined
	$4\sinh^3\theta+3\sinh\theta=$ $\frac{4}{8}\left(e^{3\theta}-3e^{\theta}+3e^{-\theta}-e^{-3\theta}\right)+\frac{1}{2}\left(3e^{\theta}-3e^{-\theta}\right)$	M1		correct expression for $\sinh\theta$ and attempt to expand $\left(e^{\theta}-e^{-\theta}\right)^3$
	$=\frac{1}{2}\left(e^{3\theta}-e^{-3\theta}\right)=\sinh 3\theta$	A1	3	**AG** identity proved
(b)	$16\sinh^3\theta+12\sinh\theta-3=0$ $\Rightarrow 4\sinh 3\theta-3=0$	M1		attempt to use previous result
	$\sinh 3\theta=\frac{3}{4}$	A1		
	$(3\theta=)\ln\left(\frac{3}{4}+\sqrt{\frac{9}{16}+1}\right)$	m1		correct ln form of $\sinh^{-1}$ for "their" $\frac{3}{4}$
	$\theta=\frac{1}{3}\ln 2$	A1	4	
(c)	$x=\sinh\theta=\frac{1}{2}\left(2^{\frac{1}{3}}-2^{-\frac{1}{3}}\right)$	M1		correctly substituting their expression for θ into $\sinh\theta$ removing any ln terms
	$2^{-\frac{2}{3}}-2^{-\frac{4}{3}}$	A1	2	
	Total		9	

(a) For **M1**, must attempt to expand $\left(e^{\theta}-e^{-\theta}\right)^3$ with at least 3 terms and attempt to add expressions for two terms on LHS.
For **A1**, must see both sides of identity connected with at least trailing equal signs.

(b) Withhold final **A1** if answer is given as $x=\frac{1}{3}\ln 2$.

Alternative: $2e^{3\theta}-2e^{-3\theta}-3=0 \Rightarrow 2e^{6\theta}-3e^{3\theta}-2=0$ *so* $(e^{3\theta}-2)(2e^{3\theta}+1)=0$
scores **M1** for $e^{k\theta}=p$ (quite generous) **A1** for $e^{3\theta}=2$ (and perhaps $e^{3\theta}=-0.5$)
then **m1** for correct ft from $e^{k\theta}=p \Rightarrow k\theta=\ln p$ and final **A1** for $\theta=\frac{1}{3}\ln 2$ and no other solutions

Q	Solution	Mark	Total	Comment
6(a)(i)	$z^n = \cos n\theta + \mathrm{i}\sin n\theta$	M1		
	$z^{-n} = \cos(-n\theta) + \mathrm{i}\sin(-n\theta)$ $= \cos n\theta - \mathrm{i}\sin n\theta$	E1		or $\frac{1}{\cos n\theta + \mathrm{i}\sin n\theta} \times \frac{\cos n\theta - \mathrm{i}\sin n\theta}{\cos n\theta - \mathrm{i}\sin n\theta} = \ldots$ shown – not just stated
	$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$	A1	3	**AG**
(ii)	$\left(z^n + \frac{1}{z^n} =\right) \quad 2\cos n\theta$	B1	1	
(b)(i)	$\left(z - \frac{1}{z}\right)^2\left(z + \frac{1}{z}\right)^2 = z^4 - 2 + \frac{1}{z^4}$	B1	1	***or*** $z^4 - 2 + z^{-4}$
(ii)	$(2\mathrm{i}\sin\theta)^2(2\cos\theta)^2 = 2\cos 4\theta - 2$	M1		using previous results
	$-16\sin^2\theta\cos^2\theta = 2\cos 4\theta - 2$ $8\sin^2\theta\cos^2\theta = 1 - \cos 4\theta$	A1cso	2	
(c)	$x = 2\sin\theta \Rightarrow \mathrm{d}x = 2\cos\theta\,\mathrm{d}\theta$	M1		$x = 2\sin\theta \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta} = k\cos\theta$
	$\int x^2\sqrt{4 - x^2}\,\mathrm{d}x = \int 16\sin^2\theta\cos^2\theta\,\mathrm{d}\theta$	A1		
	$= \int (2 - 2\cos 4\theta)(\mathrm{d}\theta)$	m1		correct ***or*** **FT** their (b)(ii) result
	$= 2\theta - \frac{1}{2}\sin 4\theta$	B1√		**FT** integrand of form $k(1 - \cos 4\theta)$
	$= \left[\pi - \frac{1}{2}\sin 2\pi\right] - \left[\frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3}\right]$			$x = 1 \Rightarrow \theta = \frac{\pi}{6}$; $x = 2 \Rightarrow \theta = \frac{\pi}{2}$;
	$= \frac{2\pi}{3} + \frac{\sqrt{3}}{4}$	A1cso	5	
	Total		**12**	

(a)(i)	May score **M1 E0 A1** if $z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta$ merely quoted and not proved. Condone poor use of brackets for **M1** but not for **A1**.
(b)(ii)	For **M1**, must use $2\mathrm{i}\sin\theta$ and "their" $2\cos\theta$ on LHS but condone poor use of brackets etc when squaring.
(c)	For **A1cso**, must simplify $\sin^{-1}1$ correctly in terms of π. Allow first **A1** for missing $\mathrm{d}\theta$ or incorrect limits used/seen, but withhold final **A1cso.**

Q	Solution	Mark	Total	Comment
7 (a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1+x}{1-x}\right)=\frac{1-x+1+x}{(1-x)^2}=\frac{2}{(1-x)^2}$	B1		ACF
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+u^2}$	M1		where $u=\frac{1+x}{1-x}$
	$\times\frac{2}{(1-x)^2}$	A1		correct unsimplified
	$=\frac{2}{(1-x)^2+(1+x)^2}=\frac{1}{1+x^2}$	A1	4	**AG** be convinced
(b)	either $\frac{\mathrm{d}}{\mathrm{d}x}(\tan^{-1}x)=\frac{1}{1+x^2}$ or $\int\frac{1}{1+x^2}\,\mathrm{d}x=\tan^{-1}x\quad(+c)$	B1		
	$\Rightarrow\tan^{-1}\left(\frac{1+x}{1-x}\right)=\tan^{-1}x+C$	M1		
	Putting $x=0$ gives $C=\tan^{-1}1=\frac{\pi}{4}$			
	$\Rightarrow\tan^{-1}\left(\frac{1+x}{1-x}\right)-\tan^{-1}x=\frac{\pi}{4}$	A1	3	**AG**
	Total		7	

(a) Alternative $\tan y=\frac{1+x}{1-x}$

$\sec^2 y\,\frac{\mathrm{d}y}{\mathrm{d}x}$ **M1** $=\frac{2}{(1-x)^2}$ **B1**

$\left(1+\left(\frac{1+x}{1-x}\right)^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}$ **A1** with final **A1** for proving given result

(b) Must see $\frac{\mathrm{d}}{\mathrm{d}x}(\tan^{-1}x)=\frac{1}{1+x^2}$ within attempt at part **(b)** to award **B1**

Q	Solution	Mark	Total	Comment
8(a)	$y=2(x-1)^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=(x-1)^{-\frac{1}{2}}$	B1		
	$1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=1+\frac{1}{x-1}$	M1		ft their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$(s=)\int_{(2)}^{(9)}\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,(\mathrm{d}x) \quad (=)$			$s=\int_2^9\sqrt{1+\frac{1}{x-1}}\,\mathrm{d}x$
	$\int_2^9\sqrt{\frac{x}{x-1}}\,\mathrm{d}x$	A1	3	(be convinced) **AG** (must have limits & dx on final line)
(b)(i)	$\cosh^{-1}3=\ln\left(3+\sqrt{8}\right)$	M1		
	$\left(1+\sqrt{2}\right)^2=3+2\sqrt{2}=3+\sqrt{8}$			need to see this line OE
	$\cosh^{-1}3=\ln\left(1+\sqrt{2}\right)^2=2\ln\left(1+\sqrt{2}\right)$	A1	2	**AG** (be convinced)
(ii)	$x=\cosh^2\theta \Rightarrow \mathrm{d}x=2\cosh\theta\sinh\theta\,\mathrm{d}\theta$	M1		$\frac{\mathrm{d}x}{\mathrm{d}\theta}=k\cosh\theta\sinh\theta$ **OE**
	$(s=)\int\frac{\cosh\theta}{\sinh\theta}2\cosh\theta\sinh\theta\,\mathrm{d}\theta$	A1		including $\mathrm{d}\theta$ on this or later line
	$2\cosh^2\theta=1+\cosh 2\theta$ **OE**	B1		double angle formula or $\frac{1}{2}\left(\mathrm{e}^{2\theta}+2+\mathrm{e}^{-2\theta}\right)$
	$(s=)\ \theta+\frac{1}{2}\sinh 2\theta$	A1		or $\left(\frac{1}{4}\mathrm{e}^{2\theta}+\theta-\frac{1}{4}\mathrm{e}^{-2\theta}\right)$
	$\left.\begin{array}{l}\cosh^{-1}3+\frac{1}{2}\sinh(2\cosh^{-1}3)\\-\cosh^{-1}\sqrt{2}-\frac{1}{2}\sinh\left(2\cosh^{-1}\sqrt{2}\right)\end{array}\right\}$	m1		correct use of correct limits
	$(s=2\ln\left(1+\sqrt{2}\right)-\ln\left(1+\sqrt{2}\right)+6\sqrt{2}-\sqrt{2}$			must see this line OE
	$=5\sqrt{2}+\ln\left(1+\sqrt{2}\right)$	A1	6	**partial AG** (be convinced)
	Total		11	
	TOTAL		75	

(b)(i) **SC1** for

$$\cosh\left(2\ln\left(1+\sqrt{2}\right)\right)=\frac{1}{2}\left(\left(1+\sqrt{2}\right)^2+\left(1+\sqrt{2}\right)^{-2}\right)=\frac{1}{2}\left(3+2\sqrt{2}+3-2\sqrt{2}\right)=3\Rightarrow\cosh^{-1}3=2\ln\left(1+\sqrt{2}\right)$$

(ii) Another possible correct form for **m1** is

$$2\ln\left(1+\sqrt{2}\right)-\ln\left(1+\sqrt{2}\right)+\frac{1}{2}\sinh\left(4\ln\left(1+\sqrt{2}\right)\right)-\frac{1}{2}\sinh\left(2\ln\left(1+\sqrt{2}\right)\right)$$